MATH 1500 D01/D02 Fall 2015

Assignment 4

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as 1/7 as opposed to 0.142857. This assignment covers sections 4.1, 4.2, 4.3, and 4.5.

This assignment is out of 60 points.
Due Date: **November 7, 2015**

1. Find the critical numbers of the function.

[4] (a) $f(x) = x^4 - 8x^2 + 10$

[4] (b) $f(x) = \dfrac{2x^2}{x+2}$

[6] 2. Find the absolute maximum and absolute minimum values of f on the given interval. Justify your answer.

$$f(x) = 2x^3 - 3x^2 - 12x + 5, \quad [-2, 4].$$

[7] 3. Verify that the following function satisfies the hypotheses of the Mean Value Theorem on the the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^3 - x^2 - x + 1, \quad [0, 2].$$

4. Consider $f(x) = x + \dfrac{25}{x}$. Note then that

$$f'(x) = \frac{x^2 - 25}{x^2} \quad \text{and} \quad f''(x) = \frac{50}{x^3}.$$

Find

[1] (a) the domain of the function.

[2] (b) the critical numbers.

[2] (c) the open intervals where the function is increasing.

[2] (d) the open intervals where the function is decreasing.

[2] (e) the x and y coordinates of any local (relative) extrema using the first derivative test.

[2] (f) the x and y coordinates of any local (relative) extrema using the second derivative test.

5. Consider the curve given by the function

$$f(x) = \frac{x(2-x)}{(x+2)^2}.$$

Then

$$f'(x) = \frac{-2(3x-2)}{(x+2)^3} \quad \text{and} \quad f''(x) = \frac{12(x-2)}{(x+2)^4}.$$

Find

[2] (a) Determine the domain of $f(x)$ and its intercepts.

[4] (b) Find all vertical asymptote(s) of $f(x)$. (Show all your work)

[4] (c) Find all horizontal asymptote(s) of $f(x)$. (Show all your work)

[2] (d) Find all critical points of $f(x)$ (that is, all critical numbers, together with their y values).

[4] (e) Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is increasing.

[2] (f) Find the x and y coordinates of any local (relative) maxima and/or local (relative) minima.

[4] (g) Determine the open intervals upon which $f(x)$ is concave up and the open intervals where $f(x)$ is concave down.

[2] (h) Find the x and y coordinates of all inflection point(s).

[4] (i) Use the above information to give a neat sketch of the graph $y = f(x)$.