Assignment 4

This assignment is based on units 7 and 8.

1.
Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\displaystyle\int_{-\infty}^{\infty} x^3 e^{-x^4}\, dx$

(b) $\displaystyle\int_{0}^{1} \frac{1}{4y-1}\, dy$

(c) $\displaystyle\int_{0}^{1} \frac{\ln x}{\sqrt{x}}\, dx$

2.
Find the length of the curve.

(a) $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}$, $\qquad 1 \le y \le 2$

(b) $x^2 = (y-4)^3$, $\qquad$ from $P(1.5)$ to $Q(8,8)$

3.
Find the area of the surface obtained by rotating the curve about the x-axis.

(a) $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$, $\qquad \frac{1}{2} \le x \le 1$

(b) $9x = y^2 + 18$, $\qquad 2 \le x \le 6$

4.
The curve $y = 1 - x^2$ with $0 \le x \le 1$ is rotated about the y-axis. Find the area of the resulting surface.