MATH 1700 D01 Summer 2015 Assignment 4

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as e^2 as opposed to a decimal approximation. Simplify your answers as much as possible.

1. Find the following limits using L'Hopital's Rule. Make sure to justify that you can use L'Hopital's Rule.

   (a) $\displaystyle\lim_{x\to 1} \frac{\ln x}{x^4 - 1}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sin 2x - 2x}{x^3}$

   (c) $\displaystyle\lim_{x\to -\infty} x \sin\bigl(1/x\bigr)$

   (d) $\displaystyle\lim_{x\to \infty} (xe^{2/x} - x)$

   (e) $\displaystyle\lim_{x\to 0^+} x^{\sqrt{x}}$

   (f) $\displaystyle\lim_{x\to 0} (e^x + x)^{4/x}$

2. Find whether the following improper integrals converge or diverge. If the integral converges, evaluate the integral. To get any credit, you must use what the improper integral means (limits) and not any shortcuts.

   (a) $\displaystyle\int_0^\infty \frac{dz}{z^2 + 3z + 2}$

   (b) $\displaystyle\int_2^\infty \frac{x^2}{x^3 - 1}\, dx$

   (c) $\displaystyle\int_4^{31} \frac{2}{\sqrt[3]{x - 4}}\, dx$

3. Use the comparison test to determine whether the following improper integrals converge.

   (a) $\displaystyle\int_1^\infty \frac{e^{-x} + 3}{x^2}\, dx$

   (b) $\displaystyle\int_0^\infty \frac{1}{x^3 + \sqrt{x}}\, dx$

4. Let R be the region below $y = \dfrac{1}{x^{2/3}}$, above the x-axis and to the right of $x = 1$. Note that this region is not bounded.

(a) Show that the area of R is unbounded.

(b) Show that the volume of R rotated about the x-axis is bounded and find the volume.

5. If $f(t)$ is continuous for $t \geq 0$, the Laplace Transform of f is the function F defined by

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt.$$

Find the Laplace Transform of $f(t) = t^2$. Hint: Use can use without proof (although the proof would just require L'Hopital's Rule) that for all real numbers n, and $s > 0$:

$$\lim_{z\to\infty} z^n e^{-sz} = 0.$$