Lesson 2: Cost & Revenue, Demand & Supply

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Find the equations of the following lines:

(a) The line with a slope of 5/3 and a y-intercept of –2.

(b) The line passing through the points (2, –3) and (6, 7).

(c) The line passing through the point (1, 1) and perpendicular to the line $2x - 7y = 5$.

(d) The line passing through the point (1, –1) and parallel to the line $y = 4x + 6$.

2. The cost C (in thousands of dollars) of a company that produces q widgets is given by $C = 12q + 40$.

(a) What is the cost of producing 50 widgets?

(b) How many widgets would cause a cost of $124,000?

3. The table below shows the number of inhabitants n (in thousands) in three cities and the amount of garbage produced each week g (in hundreds of metric tons).

n	20	25	40
g	17	35	89

(a) Does this data show a linear trend?

(b) Use this data to state g as a function of n.

4. In 1990, a company's sales were 20 million dollars. In 2000, they were 27 million dollars. Assuming the trend is linear, predict the sales in 2003.

5. A theatre has a fixed cost of \$3,000 per day and a variable cost of \$2 per customer. The admission fee is \$6 per customer.

(a) Find the cost and revenue functions. How many customers are needed to break even?

(b) Find the profit function and illustrate the break even point calculated in part (a) by sketching a graph of the profit function.

(c) What is the marginal cost, marginal revenue and marginal profit?

6. Let the demand curve for a certain product be $2p+q=100$ and the supply curve be $3p-q=50$ where p is the price in dollars and q is the quantity produced or sold. Find the equilibrium price. Illustrate this graphically.

7. The cost of producing widgets is $C(x)=x^2+8x+5$ where C is in dollars and x is measured in hundreds of widgets. Each widget sells for fourteen cents.

(a) How many widgets must be produced to break-even? Illustrate this point or points by graphing the cost and revenue functions.

(b) Find the profit in producing 200 widgets.

(c) Find the maximum profit and the production level that will attain it.

Equations of Lines

$y = mx + b$ is the equation of a straight line.

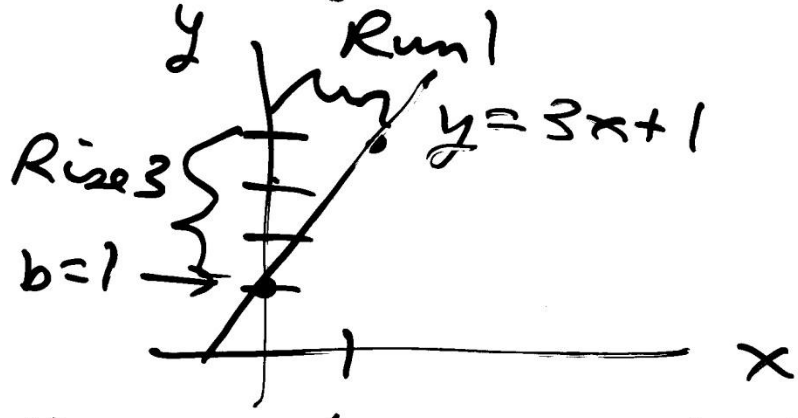
m = the slope of the line $\left(= \frac{\text{Rise}}{\text{Run}}\right)$

b = the y-intercept (where the line crosses the y-axis)

eg. $y = 3x + 1 \rightarrow m = 3 = \frac{3}{1}$

$b = 1$

To graph this line plot "b" on the y-axis then count $\frac{\text{Rise}}{\text{Run}}$ from b to get a 2nd point.



$m = \frac{\text{Rise}}{\text{Run}} = \frac{3}{1}$

Up 3 over 1

We can, of course, graph any line by simply plotting 2 points.

To get the equation of a line we need:
A point on the line (x_0, y_0)
and the slope of the line "m".
We can then use the point-slope formula to get the equation:

$$\boxed{y - y_0 = m(x - x_0)}$$ Memorize

We then can rearrange this formula into one of these forms:

1. Slope-intercept form

$$\boxed{y = mx + b}$$ (b is the y-intercept)

2. Standard form

$$\boxed{ax + by = c}$$ *(This form is rarely used in this course.)*

Lecture Problems:

1.(a) Given: $m = \frac{5}{3}$ and $b = -2$

This is perfect for $y = mx + b$ form!

Answer: $\boxed{y = \frac{5}{3}x - 2}$

To convert to standard form, simply move the x term to the LHS

$$\boxed{-\frac{5}{3}x + y = -2}$$

Traditionally, we remove fraction when using standard form, so multiply every term by 3:

$$\cancel{3}\left(-\frac{5}{\cancel{3}}x\right) + 3(y) = 3(-2) \rightarrow \boxed{-5x + 3y = -6}$$

1.(b) Given: points $(\underset{x_1}{2}, \underset{y_1}{-3})$ and $(\underset{x_2}{6}, \underset{y_2}{7})$

Get the slope: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{7 - (-3)}{6 - 2} = \frac{10}{4} = \frac{5}{2}$$

Use point-slope formula: $y - y_1 = m(x - x_1)$

$$y - (-3) = \frac{5}{2}(x - 2) \rightarrow y + 3 = \frac{5}{2}x - 5$$

$\boxed{y = \frac{5}{2}x - 8}$ OR $-\frac{5}{2}x + y = -8 \rightarrow \boxed{-5x + 2y = -16}$

1.(c) Note: If 2 lines (L_1 and L_2) are perpendicular, then their slopes (m_1 and m_2) are negative reciprocals.

ie. $\boxed{\text{If } L_1 \perp L_2 \text{, then } m_1 = \frac{-1}{m_2}}$

Given: point $(\underset{x_1}{1}, \underset{y_1}{1})$ and the line $2x - 7y = 5$

Convert the line to $y = mx + b$ form in order to read off m:

$2x - 7y = 5 \rightarrow -7y = -2x + 5 \rightarrow$ divide by -7

$\frac{\cancel{-7}y}{\cancel{-7}} = \frac{-2x}{-7} + \frac{5}{-7} \rightarrow \underbrace{y = \underset{m}{\frac{2}{7}} \underset{x}{x} - \underset{+b}{\frac{5}{7}}}$

This line has $m = \frac{2}{7}$; our line is perpendicular, so its slope is $m = -\frac{7}{2}$ (the negative reciprical)

Thus: $(x_1, y_1) = (1, 1)$; $m = -\frac{7}{2}$

$y - y_1 = m(x - x_1) \rightarrow y - 1 = -\frac{7}{2}(x - 1)$

$y - 1 = -\frac{7}{2}x + \frac{7}{2}{+1}$ (Note: $\frac{7}{2} + 1 = \frac{7+2}{2} = \frac{9}{2}$)

$\boxed{y = -\frac{7}{2}x + \frac{9}{2}}$ OR $\frac{7}{2}x + y = \frac{9}{2} \rightarrow \boxed{7x + 2y = 9}$

1.(d) Given: $\underset{x_1}{(1},\underset{y_1}{-1)}$ and parallel line $y = 4x + 6$

If 2 lines are parallel they have the same slope. ie. $L_1 \parallel L_2$ then $m_1 = m_2$

Given $y = \underset{m}{4}x + \underset{b}{6} \rightarrow m = 4$

$(x_1, y_1) = (1, -1)$

$y - y_1 = m(x - x_1) \rightarrow y - (-1) = 4(x - 1)$

$y + 1 = 4x - 4 \rightarrow \boxed{y = 4x - 5}$

OR $\boxed{-4x + y = -5}$

2.(a) $C = 12q + 40$

$q = 50 \rightarrow C = \underbrace{12(50)}_{600} + 40 = 640$

The cost is 640 thousand dollars.

2.(b) $C = 124$ (C is in thousands!)

$124 = 12q + 40$

$84 = 12q \rightarrow \frac{\cancel{12}q}{\cancel{12}} = \frac{84}{12} = 7$

7 widgets would cost $124,000.

3.

n	20	25	40
g	17	35	89

3.(a) We have been given three points

$(20, 17)$ $(25, 35)$ $(40, 89)$
$x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$

If this is linear, the slope of any 2 of these points should be the same. Use the first point as an "anchor": Find "m" for that first point together with all the other points. m must be the same every time to be linear.

1st & 2nd point: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{35-17}{25-20} = \frac{18}{5}$

1st & 3rd: $m = \frac{y_3 - y_1}{x_3 - x_1} = \frac{89-17}{40-20} = \frac{72 \div 4}{20 \div 4} = \frac{18}{5}$ ✓

Yes, this data is linear (all 3 points fall on the same line).

3.(b) g as a function of n

Note: $y = mx + b$ states y as a function of x (y depends on x)

Since y is isolated, we are giving y as a function of x.

eg. $y = 2x + 3$

In problem 2, $C = 12q + 40$

C is a function of q.

g is sort of our y

n is sort of our x

$(x_1, y_1) = (\underset{n}{20}, \underset{g}{17})$; $m = \frac{18}{5}$

$y - y_1 = m(x - x_1) \rightarrow y - 17 = \frac{18}{5}(x - 20)$

$y - 17 = \frac{18}{5}x - \frac{36\cancel{0}}{\cancel{5}}\,72$

$\underset{g}{y} = \frac{18}{5}\underset{n}{x} - 55$

$\therefore \boxed{g = \frac{18}{5}n - 55}$

Don't change to standard form

This gives g as a function of n

4. Predict the sales → "y"

$y = mx + b$ is perfect for predicting y

Let x = the year, y = the sales (in millions \$)

Given: $(1990, 20)$ and $(2000, 27)$
$\quad\;\; x_1 \quad y_1 \qquad\quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{27 - 20}{2000 - 1990} = \frac{7}{10} = m$$

$$y - y_1 = m(x - x_1) \rightarrow y - 20 = \frac{7}{10}(x - 1990)$$

$$y - 20 = \frac{7}{10}x - \frac{7(199\cancel{0})}{1\cancel{0}}$$

Wait a minute, it would have been easier to use the point $(2000, 27)$
$\qquad x_1 \quad y_1$

$$y - 27 = \frac{7}{10}(x - 2000) \rightarrow y - 27 = \frac{7}{10}x - \underbrace{\frac{1400\cancel{0}}{1\cancel{0}}}_{-1400+27}$$

$$y = \frac{7}{10}x - 1373$$ predicts the sales for any year "x"

$$x = 2003 \rightarrow y = \frac{7}{10}(2003) - 1373$$

STOP HERE if time is an issue

Sales will be $\frac{7}{10}(2003) - 1373$ million dollars

(ie $\frac{14021}{10} - 1373 = 1402.1 - 1373 = 29.1$ million dollars
$\qquad\qquad\qquad\qquad\quad\;\; 1373$

In word problems they will often give you a fixed # and a variable rate.

eg. The cost of manufacturing a product has a fixed cost of \$500 and has a variable cost of \$10 per unit produced.

→ fixed cost = b, the y-intercept

variable cost = m, the slope

x = # of units produced.

cost per unit

Let C = the cost (\$)

$$y = mx + b$$

$$C = 10x + 500$$

> Note: The Fixed cost is the cost of producing $x = 0$ units!

Cost and Revenue Problems

C = Cost = amount of money ($) we spend in producing a product

R = Revenue = amount of money ($) we get for selling the product

Breakeven Point

Revenue = Cost

Profit = Revenue − Cost

5.(a) fixed cost of \$3000 → b

variable cost of \$2 per customer → m

x = # of customers

$y = mx + b$

$$\boxed{C = 2x + 3000}$$

OR Function Notation

$$C(x) = 2x + 3000$$

Revenue: \$6 per customer

(variable revenue → m = 6)

No fixed revenue → b = 0

$y = mx + \cancel{b}$

$$\boxed{R = 6x} \text{ OR } R(x) = 6x$$

Breakeven? Revenue = Cost

$R = 6x$

$C = 2x + 3000$ → Set $R = C$

$6x = 2x + 3000$

$4x = 3000$

$x = \frac{3000}{4} = \frac{1500}{2} = 750$

We need 750 customers to break even.

5.(b) Profit = Revenue − Cost

$P = R - C$

$P = 6x - (2x + 3000)$ Brackets!

$\boxed{P = 4x - 3000}$ is the profit function

Plot 2 points

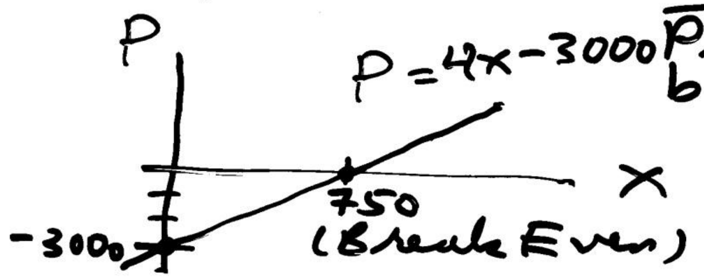
→ Plot intercept → $b = -3000$

→ Plot Breakeven point → $x = 750$

x	P
750	0

→ $P = 0$ b/c it is Breakeven

Profit = 0 when you breakeven



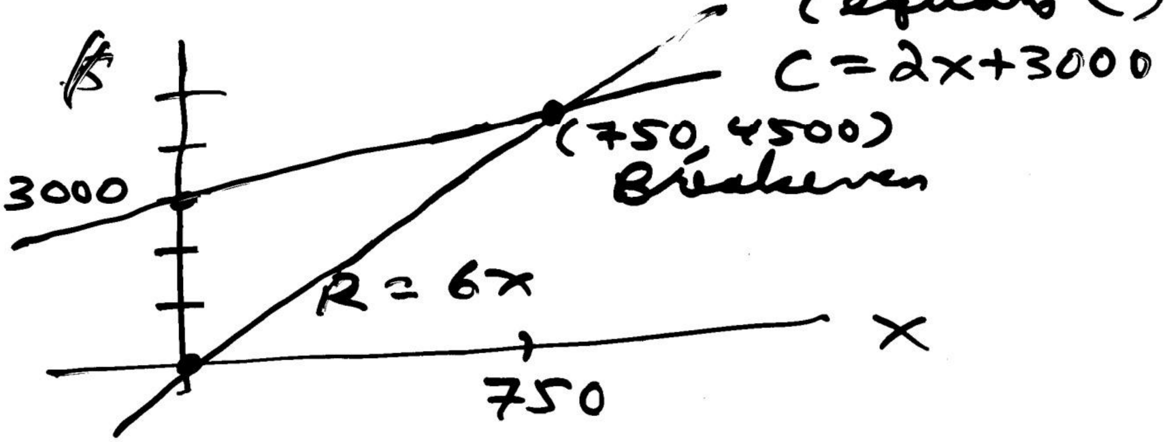
Let's Graph the Cost & Revenue Lines and illustrate the Breakeven point. $\boxed{\text{Cost = Revenue when the 2 lines cross}}$

$C = 2x + 3000$, $R = 6x$

$x = 750$ is breakeven

$\underline{C}$: $b = 3000$; $x = 750$, $C = 4500$

R: $b = 0$; $x = 750$, $R = 4500$ ✓ (equals C)



5.(c) Marginal Cost (MC) is the rate of change of the cost.

MC is the cost of producing the "next unit".

MC is the slope of the Cost line

Similarly Marginal Revenue (MR) is the expected revenue for the "next unit" sold

MR is the slope of the Revenue line

Marginal Profit (MP or $M\pi$) is the slope of the profit line (expected profit from selling the next unit.)

We have

$$C = 2x + 3000 \rightarrow m = 2$$

$$R = 6x \rightarrow m = 6$$

$$\pi = 4x - 3000 \rightarrow m = 4$$

Marginal Cost = \$2 per customer

(marginal values are \$ per unit)

Marginal Revenue = \$6 per customer

Marginal Profit = \$4 per customer

Demand and Supply

The Demand function relates the quantity of units "q" consumers will buy at a set price per unit "p".

The Supply function relates the quantity of units "q" producers are willing to make at a set price per unit "p".

Typically, high prices cause demand to drop (People did not buy DVD players when they cost $1000, but, when the price came down, many more were purchased.)

Conversely, producers want high prices (There is very little incentive for a farmer to plant wheat if he will get a low price for it.)

The goal is to reach Equilibrium, where Demand = Supply.

Equilibrium occurs when, at a specific price, producers are willing to supply exactly as many units as consumers demand.

6.

Demand: $2p + q = 100$
Supply: $3p - q = 50$

Solve these 2 equations to find where Demand = Supply

Adding: $5p = 150$

$p = 30$

$\boxed{\text{The equilibrium price is \$30 per unit.}}$

Alternate Method:

We could isolate q in both equations (Expressing q as a function of p.)

Demand: $2p + q = 100 \rightarrow \boxed{q = 100 - 2p}$

Supply: $3p - q = 50 \rightarrow \boxed{q = 3p - 50}$

Now, since both equations equal "q", we can set them equal to each other:

Demand = Supply $\rightarrow 100 - 2p = 3p - 50$

$150 = 5p$

$p = 30$ ✓

The Equilibrium point is where the Demand and Supply lines intersect.

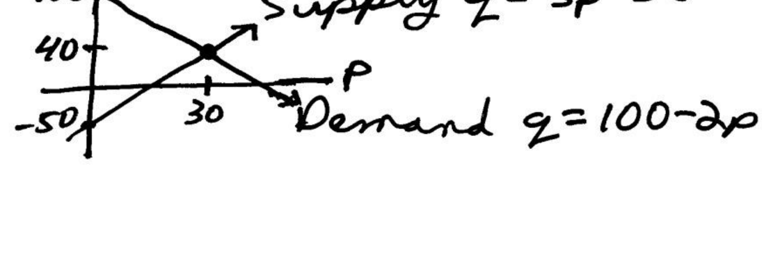
Feed $p = 30$ into either line to get "q":

Demand: $q = 100 - 2p \rightarrow p = 30,\ q = 100 - 60 = 40$

(Supply, $q = 3p - 50$, also gives $q = 40$).

Note: $q = 100 - 2p \rightarrow q = -2p + 100 \rightarrow$ y-intercept $= 100$

$q = 3p - 50 \rightarrow$ y-intercept $= -50$



7. $C = x^2 + 8x + 5$ "x" is in hundreds of widgets: 1 widget sells for 14¢

100 widgets sells for $14¢ \cdot 100 = \$14$

Revenue is \$14 for 100 widgets "x"

$\therefore$ $R = 14x$ (note this is a line)

7.(a) Break-even is $R = C$

$14x = x^2 + 8x + 5 \rightarrow x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

Break-even when $x = 5$ OR $x = 1$

(i.e. If we make and sell 100 widgets OR 500 widgets.)

$C = x^2 + 8x + 5$ is a parabola

$a = 1, b = 8, c = 5 \rightarrow$ Vertex: $x = \frac{-b}{2a} = \frac{-8}{2} = -4$

$C = (-4)^2 + 8(-4) + 5 = -11 \rightarrow (-4, -11)$

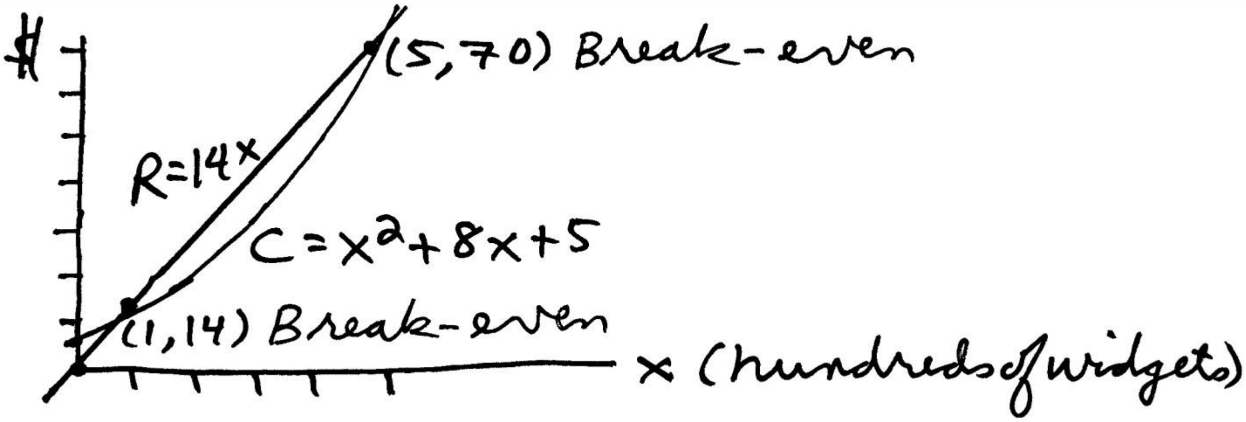
We should also plot $x = 1$ and $x = 5$ since they are the Break-even points.

Let's also plot $x = 0$ since $x \geq 0$.

(You can't make less than 0 widget)

x	$R = 14x$	$C = x^2 + 8x + 5$
0	0	5
1	14	14 ✓ (R=C)
5	70	70 ✓ (R=C)

It's not worth plotting the Vertex since $(-4, -11)$ is meaningless in this context $(x \geq 0)$.



7.(b) Profit = Revenue − Cost

$P = 14x - (x^2+8x+5)$

$P = -x^2+6x-5$ parabola

200 widgets means $x = 2$!!

$P = -(2)^2+6(2)-5 = 3$

The profit is \$3 on 200 widgets.

7.(c) $P = -x^2+6x-5 \rightarrow a=-1, b=6, c=-5$

Parabola opens down ($a = \ominus 1$), so the Vertex is the Max Value of "y" (ie P)

<u>Vertex:</u> $x = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$

$P = -(3)^2+6(3)-5 = 4$

$\Big\}$ $(3,4)$

The maximum profit is \$4 at a production level of 300 widgets.

Homework:

- ➲ Study the lesson thoroughly until you can do all of **questions 1 to 7** on pages 25 and 26 from start to finish without any assistance.
- ➲ Do all of the **Practise Problems** below (solutions are on pages 46 to 48).

Practise Problems:

1. Producing x cars costs $C(x) = 10x + 150$ thousand dollars. Each car is sold for 20 thousand dollars.

(a) Find the revenue function $R(x)$.

(b) How many cars must be sold to break even?

(c) What is the profit if 50 cars are sold?

(d) How many must be sold for a profit of $400,000?

(e) Sketch $C(x)$ and $R(x)$ on the same graph.

2. A firm producing the Latest Craze kid's doll finds the total cost of producing and selling x dolls is given by $C(x) = 20x + 3600$. They will charge $60 per doll.

(a) How many dolls must be sold to break even?

(b) What is the profit if 100 dolls are sold?

(c) How many must be sold to produce a profit of $10,000?

(d) What is the average cost per doll if 50 are produced?

3. A factory produces radios. The cost of producing x radios is $C(x) = 13x + 2400$ dollars, and they are sold for $25 each.

(a) What is the marginal cost?

(b) Find the revenue (income) function $R(x)$. What is the break-even point?

(c) Find the profit function $P(x)$. What is the marginal profit?

4. A small company produces doohickeys. It costs $450 to produce 5 doohickeys, and for each additional doohickey, the total cost increases $30.

(a) Find a linear function $C(x)$ for producing x doohickeys.

(b) If the selling price is $45 per doohickey, find the revenue function $R(x)$.

(c) Find the break-even quantity if all doohickeys produced are sold.

5. A small firm produces paperweights. The first paperweight costs \$25 to produce, and each additional paperweight costs \$5 more.

(a) Find the linear cost function $C(x)$ for producing x paperweights.

(b) If the price is \$9 per paperweight, find the revenue function.

(c) Find the **break-even** quantity, if all paperweights produced are sold.

(d) How many paperweights must be sold to make a profit of \$1000?

6. A new business makes and sells shovels. It cost \$250 in total to produce the first fifteen shovels. It cost \$450 in total to produce the first thirty-five shovels.

(a) Assuming it is linear, find the cost function $C(x)$ for producing x shovels.

(b) If the price is \$30 per shovel, find the revenue function.

(c) Find the **break-even** quantity, if all shovels produced are sold.

7. A demand curve is given by $75p+50q=3125$, where p is the price of the product, in dollars, and q is the quantity demanded at that price. The supply curve is given by $-75p+25q=775$, where p is the price of the product (in \$) and q is the quantity producers would supply under that price.

(a) How many items would consumers buy if the price is 3 dollars per item?

(b) How many items would producers supply under this same price?

(c) What are the equilibrium quantity and the equilibrium price?

8. Certain gadgets sell for \$450 per unit. The total cost (in dollars) of producing x units is given by $C(x)=10{,}000+3x^2$. What quantity of gadgets should be produced to maximize profit? Justify your answer.

9. Assume the revenue from selling x thingamajigs is given by $R(x)=60x-x^2$ hundred dollars. The cost to produce x units of thingamajigs is given by $C(x)=10x+400$ hundred dollars.

(a) Sketch a graph of both functions.

(b) Find a formula for the profit, $P(x)$.

(c) Find the maximum possible revenue, and the production level that yields it.

(d) Find the maximum possible profit, and the production level that yields it.

10. When the price for a certain product is \$25 per unit, the demand is 1000 units. If the price increases \$5 per unit, demand drops 5 units.

(a) Find a linear equation expressing the price p as a function of the demand q.

(b) It is known that the supply function for the above product is $S(q)=q^2-q-7075$. Find the break-even (*sic*) demand.

(c) Use the result from (a) to find the revenue function $R(q)$.

1. (a) $\boxed{R(x)=20x \text{ thousand dollars.}}$

1. (b) Break-even occurs when $R(x)=C(x)$:

$$20x=10x+150 \rightarrow 10x=150 \rightarrow \boxed{x=\frac{150}{10}=15}$$

They must sell 15 cars to break even.

1. (c) *Profit = Revenue – Cost* $\rightarrow$ $P(x)=R(x)-C(x)$:

$$P(x)=20x-(10x+150) \rightarrow \boxed{P(x)=10x-150}$$

$$P(50)=10(50)-150=500-150=\boxed{350}$$

The profit if 50 cars are sold is $350,000.

1. (d) Remember: we are using <u>thousands</u> of dollars for our units. Set $P(x)$ equal to 400 thousand dollars <u>not</u> 400,000!

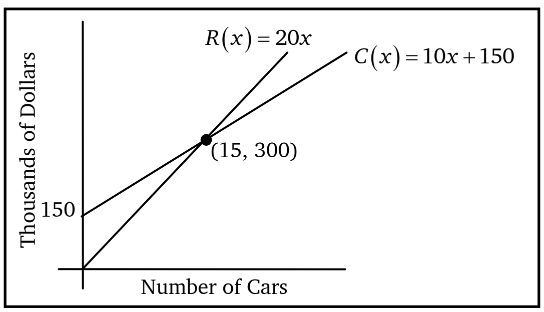
$$P(x)=10x-150=400 \rightarrow 10x=550 \rightarrow \boxed{x=\frac{550}{10}=55}$$

To make a profit of $400,000 sell 55 cars.

1. (e) For $C(x)=10x+150$, clearly the y-intercept is 150. For a second point, the best choice is the break-even point found in (b) above:

When $x=15: C(15)=10(15)+150=300$ $\rightarrow$ Plot (15, 300).

For $R(x)=20x$, clearly the y-intercept is 0 and it also passes through the point (15, 300), since the **Cost and Revenue graphs always intersect at the break-even point.** Thus:



DON'T FORGET: LABEL YOUR AXES WHEN DRAWING GRAPHS.

2. (a) Clearly: $R(x)=60x$ $\rightarrow$ Set $R(x)=C(x)$:

$$60x=20x+3600 \rightarrow 40x=3600 \rightarrow \boxed{x=\frac{360\cancel{0}}{4\cancel{0}}=90}$$

They must sell 90 dolls to break even.

2. (b) *Profit = Revenue – Cost* $\rightarrow$ $P(x)=R(x)-C(x)$:

$$P(x)=60x-(20x+3600) \rightarrow \boxed{P(x)=40x-3600}$$

$$P(100)=40(100)-3600=4000-3600=\boxed{400}$$

If 100 dolls are sold, the profit is $400.

2. (c)

$$P(x)=40x-3600=10{,}000 \rightarrow 40x=13{,}600 \rightarrow \boxed{x=\frac{13{,}60\cancel{0}}{4\cancel{0}}=340}$$

To produce a profit of $10,000, they must sell 340 dolls.

2. (d) $\textit{average cost}=\dfrac{\textit{total cost of production}}{\textit{number of items produced}}=\dfrac{C(x)}{x}$

$$\boxed{x=50}: C(50)=20(50)+3600=1000+3600=\boxed{4600}$$

It costs $4600 to produce 50 dolls: $\textit{average cost}=\dfrac{460\cancel{0}}{5\cancel{0}}=\boxed{92}$

The average cost if 50 are produced is $92 per doll.

3. (a) Recall: marginal cost is the *slope* of the cost line. By $y=mx+b$ form: $C(x)=\underset{m}{13}x+\underset{b}{2400}$, we see that the slope, m=13.

The marginal cost is $13 per radio.

3. (b) The radios sell for $25 each: $R(x)=25x$.

Break-even occurs when $R(x)=C(x)$:

$$25x=13x+2400 \rightarrow 12x=2400 \rightarrow \boxed{x=\frac{2400}{12}=200}$$

They must sell 200 radios to break even.

3. (c) *Profit = Revenue – Cost* $\rightarrow$ $P(x)=R(x)-C(x)$:

$$P(x)=25x-(13x+2400) \rightarrow \boxed{P(x)=\underset{m}{12}x-2400}$$

The marginal profit is the *slope*, m = 12.

The profit function is $P(x)=12x-2400$ **and the marginal profit is $12 per radio.**

4. (a) It costs $450 to produce 5 doohickeys: When x=5, C=450. i.e. We have been given the **point (5, 450)**.
For each additional doohickey, the total cost increases $30. This gives us the *variable cost* $\rightarrow$ the **slope *m*=30**.
Use the point-slope formula:

$$y-\underset{450}{y_0}=\underset{30}{m}(x-\underset{5}{x_0}) \rightarrow y-450=30(x-5)$$

$$y-450=30x-150 \rightarrow \boxed{y=30x+300}$$

But y is the cost, $C(x)$, therefore:

The cost function is $C(x)=30x+300$ **.**

4. (b) The selling price is $45 per doohickey: $\boxed{R(x)=45x}$.

4. (c) Break-even when $R(x)=C(x)$:

$$45x=30x+300 \rightarrow 15x=300 \rightarrow \boxed{x=\frac{300}{15}=20}$$

They must sell 20 doohickeys to break even.

5. (a) The first paperweight costs $25 to produce: When x=1, C=25. i.e. We have been given the **point (1, 25)**.
Each additional paperweight costs $5 more. This gives us the *variable cost* $\rightarrow$ the **slope *m*=5**.
Use the point-slope formula:

$$y-\underset{25}{y_0}=\underset{5}{m}(x-\underset{1}{x_0}) \rightarrow y-25=5(x-1)$$

$$y-25=5x-5 \rightarrow \boxed{y=5x+20}$$

But y is the cost, $C(x)$, therefore:

The cost function is $C(x)=5x+20$ **.**

5. (b) The price is $9 per paperweight: $\boxed{R(x)=9x}$

5. (c) Break-even when $R(x)=C(x)$:

$$9x=5x+20 \rightarrow 4x=20 \rightarrow \boxed{x=\frac{20}{4}=5}$$

They must sell 5 paperweights to break even.

5. (d) *Profit = Revenue – Cost* $\rightarrow$ $P(x)=R(x)-C(x)$:

$$P(x)=9x-(5x+20) \rightarrow \boxed{P(x)=4x-20}$$

Set $P(x)$ equal to $1000 and solve for x:

$$P(x)=4x-20=1000 \rightarrow 4x=1020 \rightarrow \boxed{x=\frac{1020}{4}=255}$$

They must sell 255 paperweights to make a $1000 profit.

6. (a) It costs \$250 in total to produce 15 shovels: when x=15, C=250. i.e. We have been given the **point (15, 250)**.
It costs \$450 in total to produce 35 shovels: when x=35, C=450. i.e. We have been given the **point (35, 450)**.
Use these two points to compute the *slope* of the line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{450-250}{35-15} = \frac{200}{20} \to \boxed{m=10}$$

Use the point-slope formula to get the cost function. Note: I am using the first point (15, 250) but you could use (35, 450) also. The answer in the end would be identical.

$$\underset{250}{y - y_0} = \underset{10}{m}(x - \underset{15}{x_0}) \to y - 250 = 10(x-15)$$

$$y - 250 = 10x - 150 \to \boxed{y = 10x + 100}$$

But y is the cost, $C(x)$, therefore:

The cost function is $C(x) = 10x + 100$.

6. (b) The price is \$30 per shovel: $\boxed{R(x) = 30x}$

6. (c) Break-even occurs when $R(x) = C(x)$:

$$30x = 10x + 100 \to 20x = 100 \to \boxed{x = \frac{100}{20} = 5}$$

They must sell 5 shovels to break even.

7. (a) The demand curve $75p + 50q = 3125$ tells us the amount q customers would buy at price p=3 dollars per item:

$$75(3) + 50q = 3125 \to 50q = 3125 - 225 = 2900 \to \boxed{q = \frac{290\cancel{0}}{5\cancel{0}} = 58}$$

If the price is 3 dollars, consumers would buy 58 items.

7. (b) The supply curve $-75p + 25q = 775$ tells us the amount q producers would supply at price p=3 dollars per item:

$$-75(3) + 25q = 775 \to 25q = 775 + 225 = 1000 \to \boxed{q = \frac{1000}{25} = 40}$$

If the price is 3 dollars, producers would supply 40 items.

7. (c) Equilibrium is when *Demand = Supply*. Solve two equations with two unknowns to find this point:

$$\left.\begin{array}{ll} \text{Demand:} & \cancel{75p} + 50q = 3125 \\ \text{Supply:} & \cancel{-75p} + 25q = 775 \end{array}\right\} \text{Add the 2 equations}$$

$$75q = 3900 \to \boxed{q = \frac{3900}{75} = 52}$$

Sub q=52 into $75p + 50q = 3125$:

$$75p + 50(52) = 3125 \to 75p = 3125 - 2600 = 525 \to \boxed{p = \frac{525}{75} = 7}$$

Equilibrium is 52 items at a price of \$7 per item.

8. Given $C(x) = 10{,}000 + 3x^2$; \$450 per gadget: $R(x) = 450x$.
Profit = Revenue – Cost → $P(x) = R(x) - C(x)$:

$$P(x) = 450x - \left(10{,}000 + 3x^2\right) \to \boxed{P(x) = \underset{a}{-3}x^2 + \underset{b}{450}x - 10{,}000}$$

$P(x)$ is a quadratic, $y = ax^2 + bx + c$, and so a parabola.

The vertex is at $x = -\frac{b}{2a} = -\frac{450}{2(-3)} = 75$ gadgets.

Since a=–3 (negative), the parabola opens down "$\cap$" making the vertex the maximum.

To maximize profit make and sell 75 gadgets.

9. (a) A good graph of both functions should show where they cross each other. Set $R(x) = C(x)$ to find the intersection (which means we are finding the break-even point actually):

$$60x - x^2 = 10x + 400$$

Pull everything to one side equal to 0 and factor:

$$x^2 - 50x + 400 = 0 \to (x-10)(x-40) = 0$$

The graphs intersect when x=10 and x=40.

$C(x) = 10x + 400$ is a **line**, plot the 2 points of intersection to graph it:

x	10	40
$C(x)$	500	800

$R(x) = 60x - x^2 = \underset{a}{-1}x^2 + \underset{b}{60}x$ is a **parabola**, Recall, for the parabola $y = ax^2 + bx + c$ the vertex is at $x = -\frac{b}{2a}$. Therefore:

$$x = -\frac{b}{2a} = -\frac{60}{2(-1)} = 30 .$$

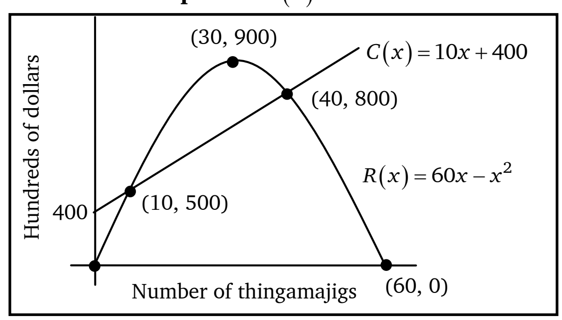
When x=30, $R(30) = 60(30) - (30)^2 = 900$.

The vertex is (30, 900).

It would also be good to compute the zeros of $R(x)$ (the x-intercepts of the parabola):

$$R(x) = 60x - x^2 = 0 \to x(60-x) = 0$$

The x-intercepts for $R(x)$ are x=0 and x=60.



9. (b) *Profit = Revenue – Cost* → $P(x) = R(x) - C(x)$:

$$P(x) = 60x - x^2 - (10x + 400) = 60x - x^2 - 10x - 400$$

$$\boxed{P(x) = -x^2 + 50x - 400}$$

9. (c) The vertex of the revenue parabola (30, 900) tells us the maximum value is 900 hundred dollars (i.e. \$90,000):

The maximum possible revenue is \$90,000 at the production level of 30 thingamajigs.

9. (d) $P(x) = -x^2 + 50x - 400 = \underset{a}{-1}x^2 + \underset{b}{50}x - 400$ is also a parabola, and so its vertex identifies the maximum possible profit:

$$x = -\frac{b}{2a} = -\frac{50}{2(-1)} = 25$$

When x=25:

$$P(25) = -(25)^2 + 50(25) - 400 = -625 + 1250 - 400 = 225 .$$

The vertex is (25, 225), telling us the profit is 225 hundred dollars (i.e. \$22,500) for 25 thingamajigs.

The maximum possible profit is \$22,500 at the production level of 25 thingamajigs.

10. When p= \$25, then q=1000 units. If the price increases \$5 per unit, demand drops 5 units. So, if we add \$5 to the price (p=\$30), then the demand will drop by 5 (q=995 units). So, we have two points then:

$$p_1=25,\ q_1=100 \text{ and } p_2=30,\ q_2=995.$$

10. (a) They want p as a function of q, so that's like y as a function of x. i.e. p is our "y" and q is our "x".

Use the two points above to compute the *slope* of the line:

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{p_2-p_1}{q_2-q_1}=\frac{30-25}{995-1000}=\frac{5}{-5}\rightarrow \boxed{m=-1}$$

Use the point-slope formula to get the demand function. Note: I am using the first point (1000, 25) but you could use (995, 30) also. The answer in the end would be identical.

$$\underset{p}{y}-\underset{25}{y_0}=\underset{-1}{m}(\underset{q}{x}-\underset{1000}{x_0})\rightarrow p-25=-1(q-1000)$$

$$p-25=-q+1000\rightarrow \boxed{p=-q+1025}$$

The demand function is $p=-q+1025$ **or** $D(q)=-q+1025$.

10. (b) I have written *sic* here because this is exactly how a question on a recent exam was phrased when this clearly is incorrect language. The question should have asked to find the <u>equilibrium</u> demand. We want demand and supply to be in equilibrium; it is cost and revenue that have a break-even point.

Again, the question is forcing us to read their mind that $S(q)=q^2-q-7075$ is giving us the supply price p in dollars per unit.

Equilibrium (not break-even!) is *Demand = Supply*

$D(q)=S(q)\rightarrow$ we found $D(q)$ in part (a)

$$\cancel{-q}+1025=q^2\cancel{-q}-7075$$

$$1025+7075=q^2\rightarrow q^2=8100$$

$$q=\pm\sqrt{8100}=\pm 90$$

The quantity cannot be negative, so q=90.

The equilibrium demand is 90 units.

10. (c) $\textit{Revenue}=\textit{price}\times\textit{quantity}\rightarrow R(q)=p\times q$.

But, from part (a), we know that $p=-q+1025$, so:

$$R(q)=p\times q=(-q+1025)\times q=-q^2+1025q$$

The revenue function is $R(q)=-q^2+1025q$.