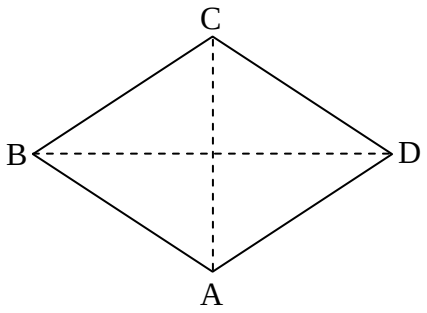
MATH 1300 ASSIGNMENT PROBLEMS (UNIT 1)

[10] ABCD is a rhombus (a parallelogram with all four sides of equal length).



(a) Write the vector $\overrightarrow{BD}$ as a linear combination of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

(b) Write the vector $\overrightarrow{AC}$ as a linear combination of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

(c) Use vector methods to show that $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{BD}$.

[10] 2. Let A = (5, 7, –4) and B = (3, –5, 1) be two points in $\mathbf{R}^3$.

(a) Find the components of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$.

(b) Find the coordinates of the point C if $\overrightarrow{AC} = \overrightarrow{CB}$.

(c) The point D = (*k*, 2, 3) is equidistant from the points A and B. Find the value(s) of *k*.

(d) Find the coordinates of the point X for which $\overrightarrow{AX} = 2\overrightarrow{AB}$.

[10] 3. Let $\mathbf{u} = (4,3,1)$, $\mathbf{v} = (5,4,-2)$ and $\mathbf{w} = (7,-3,4)$ be three vectors in $\mathbf{R}^3$. Find the following.

(a) $2\mathbf{u} - 3\mathbf{v} + \mathbf{w}$

(b) $\mathbf{u} \bullet \mathbf{v}$

(c) $\mathbf{v} \times \mathbf{u}$

(d) $\text{proj}_{\mathbf{v}}\mathbf{u}$

(e) sine of the angle between the vectors **u** and **v**.

[10] 4. Let **u** = (4, 6, 3) and **v** = (2, 3, *k*) be two vectors in $\mathbf{R}^3$.

(a) For what value(s) of *k* will the two vectors **u** and **v** be parallel? Explain.

(b) For what value(s) of *k* will the two vectors **u** and **v** be orthogonal? Explain.

(c) For what value(s) of k will the two vectors **u** and **v** be of equal length?

[10] 5. Let l: $2x + 3y = 6$ be a line and P = (5, 3) be a point in $\mathbf{R}^2$.

(a) Let Q be the point on the line l having its y-coordinate = 0 and let R be the point on the line l having its x-coordinate = 0. Find the coordinates of the points Q and R.

(b) Plot the points P, Q, R and the line l on a two-dimensional Cartesian coordinate system.

(c) Find the components of the vector $\overrightarrow{QP}$.

(d) Find a normal vector **n** to the given line l.

(e) Find the distance between the point P and the line l.

[10] 6. The plane $x + 2y + 2z = 4$ intersects the positive coordinate axis OX, OY and OZ in three points A, B and C respectively.

(a) Find the coordinates of the three points A, B and C.

(b) Find the area of the triangle ABC.