# MATH 1300 ASSIGNMENT 2 PROBLEMS (UNIT 2)

[10] 1. Let P = (2, 3, 1), Q = (4, 1, 2) and R = (1, 2,-3) be 3 points in $\mathbf{R}^3$.

(a) Find the components of the vector $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(b) Find a set of parametric equations for the line through the points P and R.

(c) Use the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ to find a normal vector to the plane through the 3 points P, Q and R.

(d) Find a standard form equation of the plane through the 3 points P, Q and R.

[10] 2. Let $\boldsymbol{\pi}_1 :\ 2x - 3y + z = 8$ and $\boldsymbol{\pi}_2 :\ 4x - y + 3z = 11$ be two planes in $\mathbf{R}^3$.

(a) Find a normal vector $\mathbf{n_1}$ to the plane $\boldsymbol{\pi}_1$ and a normal vector $\mathbf{n_2}$ to the plane $\boldsymbol{\pi}_2$.

(b) Find the cosine of the dihedral angle between the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$.

(c) Find a vector $\mathbf{v}$ parallel to the line of intersection of the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$.

(d) Find the point on the line of intersection of the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ whose y-coordinate is 0.

(e) Use the results from parts (c) and (d) to find a set of parametric equations of the line of intersection of the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$.

[10] 3. Let $l :\ (x, y, z) = (2, -1, 3) + t(2, 1, 1)$ be a line in $\mathbf{R}^3$ and let $\boldsymbol{\pi} :\ 2x - y - 3z = 4$ be a plane in $\mathbf{R}^3$.

(a) Find a vector $\mathbf{v}$ parallel to line $l$ and a vector $\mathbf{n}$ that is normal to the plane $\boldsymbol{\pi}$.

(b) Show that the line $l$ is parallel to the plane $\boldsymbol{\pi}$.

(c) Set $t = 0$ in the vector equation for the line $l$ to find a point P on the line $l$. Set $y = z = 0$ to find a point Q on the plane $\boldsymbol{\pi}$. Find the components of the vector $\overrightarrow{QP}$.

(d) Find the distance between the line $l$ and the plane $\boldsymbol{\pi}$.

(e) Find the point of intersection of the line $(x, y, z) = (1, 2, -2) + t(1, 1, -3)$ and the plane $\boldsymbol{\pi}$.

[10] 4. Given the skew lines $l_1: \; x = 2 - 3t, \;\; y = 1 + 4t, \;\; z = 2 + 2t$ and
$l_2: \; x = 1 - 2s, \;\; y = 2 - 3s, \;\; z = 3 + 2s$, find the following.

(a) A vector $\mathbf{v_1}$ parallel to line $l_1$ and a vector $\mathbf{v_2}$ parallel to line $l_2$.

(b) A vector **n** that is orthogonal to both lines $l_1$ and $l_2$.

(c) Sine of the angle between the lines $l_1$ and $l_2$.

(d) A point P on line $l_1$ and a point Q on line $l_2$. Find also the vector $\overrightarrow{PQ}$.

(e) The distance between the lines $l_1$ and $l_2$.

[10] 5. Given the point P = (1, 4, 2) and the plane $2x - y + 3z = 9$, find the following.

(a) A set of parametric equations for the line through P that is also orthogonal to the given plane.

(b) The point of intersection of the line from part (a) with the given plane.

[10] 6. Given $\boldsymbol{\pi}_1: \; 2x + ay + 4z = 8$ and $\boldsymbol{\pi}_2: \; ax + 4y + 8z = 18$ are standard form equations of two planes in $\mathbf{R}^3$, find the following.

(a) A normal vector $\mathbf{n_1}$ to the plane $\boldsymbol{\pi}_1$ and a normal vector $\mathbf{n_2}$ to the plane $\boldsymbol{\pi}_2$.

(b) For what value(s) of $a$ are these two planes parallel to each other?

(c) For what value(s) of $a$ are these two planes perpendicular to each other?

(d) If $a = 1$, these two planes intersect each other. Find the cosine of the dihedral angle between the two planes when $a = 1$.