MATH 1300 ASSIGNMENT PROBLEMS (UNIT 3)

[10] 1. Determine into which of the following 3 types (A), (B) or (C) the matrices (a) to (e) below can be classified.

Type (A): The matrix is in both reduced row-echelon form and row-echelon form.
Type (B): The matrix is in row-echelon form but not in reduced row-echelon form.
Type (C): The matrix is in neither reduced row-echelon form nor in row-echelon form.

(a) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 4 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(f) Find all 2×3 reduced row-echelon matrices having a bottom row containing only zeros.

[10] 2. Consider the following system of linear equations.

$$\begin{aligned} x_1 + x_3 + x_4 &= 2 \\ x_1 + x_2 - x_3 &= 6 \\ x_2 + x_3 + x_4 &= 3 \\ x_1 - x_2 + x_4 &= 0 \end{aligned}$$

(a) Write out the augmented matrix for this system of linear equations.

(b) Use elementary row operations to reduce the augmented matrix to reduced row-echelon form.

(c) Write out the solution to the system of linear equations.

[10] 3. The augmented matrix from a system of linear equations has the following reduced row-echelon form.

$$\begin{bmatrix} 1 & -3 & 0 & 2 & 0 & -1 & 0 & 7 \\ 0 & 0 & 1 & -2 & 0 & 4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) How many equations are there in the system?

(b) How many variables are there in the system?

(c) How many of the variables are independent variables?

(d) Write out the solution set for the system.

[10] 4. Consider the system of linear equations

$$\begin{aligned} x + ay &= 1 \\ 2x + 8y &= b \end{aligned}$$ where a and b are real numbers.

(a) Write out the augmented matrix for this system of linear equations.

(b) Use elementary row operations to reduce the augmented matrix to row-echelon form.

(c) Determine for what values of a and b does the system have infinitely many solutions.

(d) Determine for what values of a and b does the system have no solution.

(e) Determine for what values of a and b does the system have an unique solution.

[10] 5. Anne, Betty and Carol went to their local produce store to buy some fruit. Anne bought one pound of apples and two pounds of bananas and paid $2.11. Betty bought two pounds of apples and one pound of grapes and paid $4.06. Carol bought one pound of bananas and two pounds of grapes and paid $4.45.

(a) Let x = price of a pound of apples, y = price of a pound of bananas and z = price of a pound of grapes. Write out 3 linear equations representing the purchases of Anne, Betty and Carol.

(b) Write out the augmented matrix for your system of 3 linear equations of part (a).

(c) Use elementary row operations to row reduce the augmented matrix of part (b) to a reduced row-echelon matrix.

(d) What is the price per pound for each of the three fruits?

[10] 6. Consider the linear equation $ax + by + cz = d \quad (d \neq 0)$ (1)
and the associated homogeneous equation $ax + by + cz = 0$. (2)

Let (x_1, y_1, z_1) be a solution to equation (1) and
let (x_0, y_0, z_0) be a solution to equation (2).

(a) Let *k* be any real number different from 1. Show that (kx_1, ky_1, kz_1) is a not a solution to equation (1).

b) Let *k* be any real number. Show that (kx_0, ky_0, kz_0) is a solution to equation (2).

(c) Let *k* be any real number. Show that $(x_1 + kx_0,\ y_1 + ky_0,\ z_1 + kz_0)$ is a solution to equation (1).