MATH 1300 ASSIGNMENT PROBLEMS (UNIT 5)

[10]   1. Use a third row cofactor expansion to evaluate the determinant of

$$A = \begin{bmatrix} 5 & 1 & 4 & 1 \\ 4 & 1 & 2 & 5 \\ 2 & 3 & 4 & 1 \\ 1 & 1 & 2 & 4 \end{bmatrix}.$$

[10]   2. Compute the determinant of $A = \begin{bmatrix} 8 & 6 & 6 & 5 \\ 2 & 4 & 6 & 2 \\ 6 & 9 & 7 & 5 \\ 5 & 6 & 3 & 1 \end{bmatrix}$ by the method of using elementary

row operations to transform the matrix A into an upper triangular matrix.

[10]   3. Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 1 \\ 1 & 2 & 3 \end{bmatrix}.$

(a) Find the matrix adj(A).

(b) Compute the matrix product $A \cdot adj(A)$.

(c) Use the information from part (b) to determine the value of det(A). Explain.

(d) Use the information from parts (a) and (c) to find the inverse matrix A^{-1}.

[10]   4. Use Cramer's rule to solve the following system of linear equations.

$$x + 3y + z = 2$$
$$3x + 2y + z = 2$$
$$4x + y + 3z = 3$$

[10]   5. Let A and B be 3×3 matrices with $\det(A) = 2$ and $\det(B) = 6$. Determine the values of the following.

(a) $\det(AB)$

(b) $\det(A^{-1}B)$

(c) $\det(A^T B^{-1})$

(d) $\det(5A)$

(e) $\det(\text{adj}(AB))$

[10]   6(a) Let $A = \begin{bmatrix} k & -3 \\ 1 & k-4 \end{bmatrix}$. For what values of k is the matrix A noninvertible?

(b) Let $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & k & 1 \\ 3 & 1 & k \end{bmatrix}$. For what values of k is the matrix B noninvertible?