

## MATH 1310 Matrices for Management and Social Sciences

### Assignment 4

- [10] 1. Let  $A$  and  $B$  be  $4 \times 4$  matrices with  $\det(A) = 5$  and  $\det(B) = 2$ . Determine (if possible) the value of the following. If it is not possible to determine the value from the given information, state your answer as **Not Possible**.
- (a)  $\det(BA^2)$
  - (b)  $\det(BA^{-1})$
  - (c)  $\det(2A + B)$
  - (d)  $\det(\text{adj}(A))$
  - (e)  $\det(C)$  where  $C$  is the matrix obtained from  $A$  by multiplying the second row of  $A$  by 3.
- [10] 2. The state of Onitoba has two types of schools, public schools and private schools. Past experience shows that 90% of the students attending a public school this year will attend a public school next year while 10% will go to a private school next year. Similarly, 80% of those attending a private school this year will attend a private school next year while 20% will go to a public school next year.
- (a) Construct the transition matrix showing the school attendance projections.
  - (b) If 70% of the students attend public schools and 30% attend private schools this year, what is the expected distribution for school attendance next year?
  - (c) What will be the expected long term steady-state distribution of the students with respect to public and private schools?
- [10] 3. Let  $A = \begin{bmatrix} 1 & 2 & k \\ 3 & 6 & k \\ k & 2 & 1 \end{bmatrix}$ . For what values of  $k$  is the matrix  $A$  non-invertible?

- [20] 4. Use row operations and their determinant properties (not the definition) to find the value of the determinant of the following matrix.

$$\begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 2 & 3 & 1 \\ 3 & 1 & 2 & 4 \\ 6 & 2 & 10 & 4 \end{bmatrix}$$

[20] 5. Let  $M = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 6 & 9 \\ 4 & 5 & 7 \end{bmatrix}$ .

- (a) Find the matrix  $\text{adj}(M)$ .  
(b) Compute the matrix product  $M \cdot \text{adj}(M)$ .  
(c) Use the information from part (b) to determine the value of  $\det(M)$ .  
(d) Use the information from parts (a) and (c) to calculate  $M^{-1}$ .  
(e) Solve the system  $MX = N$  where  $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$  and  $N = \begin{bmatrix} 42 \\ 21 \\ -42 \end{bmatrix}$ .

- [10] 6. Use Cramer's rule to solve the following system of linear equations.

$$2x - y + z = 6$$

$$4x + z = 13$$

$$3x + y + z = 14$$