Matrices for Management and Social Sciences

Assignment 5

[6] 1. Find a vector equation and a set of parametric equations of the line passing through the points $P(-1, 3, 2)$ and $Q(-4, 3, 1)$.

[9] 2. Let P(3, 2, 5), Q(4, 1, 7) and R(2, 1, 4) be 3 points in $\square^3$. Find the following.

(a) $\overrightarrow{PQ} + 3\,\overrightarrow{PR}$

(b) The coordinates of the point S if $\overrightarrow{PS} = \overrightarrow{QR}$

(c) $\left|\overrightarrow{PR} + \overrightarrow{QR}\right|$

[10] 3. Let $A = \begin{bmatrix} 2 & 4 & -5 \\ 1 & 3 & -3 \end{bmatrix}$. Which of the following vectors belongs to the null space of A?

$\mathbf{u_1} = (1,2,3), \quad \mathbf{u_2} = (3,1,2), \quad \mathbf{u_3} = (0,0,0), \quad \mathbf{u_4} = (0,0), \quad \mathbf{u_5} = (6,2,4)$

[10] 4. Find the null space of the given matrix. If the null space is more than the zero vector alone, describe it as the span of a set of vectors.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

[15] 5. Determine whether or not the set W is a subspace of the given vector space V.

(a) $W = \{(b-a,b) : a,b \text{ real numbers}\}; \quad V = \square^2$;

(b) $W = \{(x,-x,2x) : x \text{ real number}\}; \quad V = \square^3$;

(c) $W = \{(1,x,0) : x \text{ real number}\}; \quad V = \square^3$.

[5] 6. Let $\mathbf{u} = (1, 2)$ and $\mathbf{v} = (-2, 1)$. Find all values of k such that $\left|\mathbf{u} + k\mathbf{v}\right| = \sqrt{30}$.

[10] 7. Find the value of k for which the following vectors are linearly dependent.

$\mathbf{u_1} = (1,k,1), \quad \mathbf{u_2} = (k+1,4,1), \quad \mathbf{u_3} = (5,7,k)$.

[15]   8. Consider the vectors $\mathbf{u_1} = (1,5,2)$, $\mathbf{u_2} = (-2,-3,3)$, $\mathbf{u_3} = (3,2,1)$, $\mathbf{w} = (0,4,4)$.

(a) Determine whether the set $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is linearly independent or dependent.

(b) Determine whether $\mathbf{w} \in \mathrm{Span}\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$.

(c) If possible, write $\mathbf{w}$ as a linear combination of $\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}$.