Assignment 4 MATH 1500

(Follows Unit 11 in the manual)

Values

[9] 1. Find the critical numbers of each of the following functions.

(a) $f(x) = x^3 - 3x$ (b) $f(x) = xe^x$ (c) $f(x) = 3x^{1/3} - x$

[12] 2. Find the absolute maximum and absolute minimum values of $f(x)$ on the given interval.
(a) $f(x) = 9x^2 - x^3$ $[-2, 10]$ (b) $f(x) = 6x^{1/3} - 2x$ $[-2, 2]$

[6] 3. Verify that the function $f(x) = 5x^3 + 3x$ satisfies the hypotheses of the Mean Value Theorem on the interval [0, 2].. Find all numbers *c* that satisfy the conclusion of the Mean Value Theorem.

[5] 4. If the Mean Value Theorem is applied to the function $f(x) = \frac{5}{2}x^{2/5}$ on the closed interval $[-1, 1]$ we conclude that there is a number *c* in the open interval $(-1, 1)$ such that $f'(c) = \dfrac{\frac{5}{2} - \frac{5}{2}}{2} = 0$. However $f'(x) = \dfrac{1}{x^{3/5}}$ is obviously never zero for any number *c* in the open interval $(-1, 1)$. Explain the contradiction.

[10] 5. Determine the interval(s) where $f(x)$ is increasing or decreasing. Find also the local maximum and local minimum values of $f(x)$.

(a) $f(x) = \dfrac{x}{(1+x)^2}$ (b) $f(x) = x^3 - 3x^2 + 5$

[8] 6. Determine where $f(x) = x^4 - 6x^3 + 1$ is concave up and where it is concave down. Find all the inflection points of $f(x) = x^4 - 6x^3 + 1$.

[20] 7. If $f(x) = \dfrac{3x^2}{x^2+5}$ $\qquad f'(x) = \dfrac{30x}{(x^2+5)^2}$ $\qquad f''(x) = \dfrac{30\left(5-3x^2\right)}{\left(x^2+5\right)^3}$

(a) Determine the intervals where *f*(*x*) is increasing and where it is decreasing.
(b) Find all local maxima and all local minima.
(c) Determine where *f*(*x*) is concave up and where it is concave down.
(d) Find all inflection points.
(e) Find all horizontal asymptotes and all vertical asymptotes.
(f) Sketch the graph of *f*(*x*). [cf. Section 4.5]

Total = 70