MATH 1500 Assignment 4

(Follows Unit 11 in the manual)

Values

[9] 1. Find the critical numbers of each of the following functions. [cf. Section 4.1]

(a) $f(x) = x^5 - 5x^3$ (b) $f(x) = x^2 e^{2x}$ (c) $f(x) = 12x^{1/3} - x$

[10] 2. Find the absolute maximum and absolute minimum values of $f(x)$ on the given interval. [cf. Section 4.1]

(a) $f(x) = 2x^3 - 15x^2$ [–2, 10] (b) $f(x) = 9x^{1/3} - 3x$ [–2, 2]

[8] 3. Verify that the function $f(x) = 5x^3 + 3x$ satisfies the hypotheses of the Mean Value Theorem on the interval [0, 2].. Find all numbers *c* that satisfy the conclusion of the Mean Value Theorem. [cf. Section 4.2]

[6] 4. Use the Mean Value Theorem to show that $\cos x \geq 1 - x$ if $x \geq 0$.

[3] 5. If the Mean Value Theorem is applied to the function $f(x) = \frac{5}{2} x^{2/5}$ on the closed interval $[-1, 1]$ we conclude that there is a number *c* in the open interval $(-1, 1)$ such that $f'(c) = \dfrac{\frac{5}{2} - \frac{5}{2}}{2} = 0$. However $f'(x) = \dfrac{1}{x^{3/5}}$ is obviously never zero for any number *c* in the open interval $(-1, 1)$. Explain the contradiction.

[10] 6. Determine the interval(s) where *f*(*x*) is increasing or decreasing. Find also the local maximum and local minimum values of *f*(*x*) . [cf. Section 4.3]

(a) $f(x) = x^5 - 15x^3 + 3$ (b) $f(x) = 3x^{4/3} - 96x^{1/3}$

[8] 7. Determine where $f(x) = x^4 - 6x^3 + 1$ is concave up and where it is concave down. Find all the inflection points of $f(x) = x^4 - 6x^3 + 1$. [cf. Section 4.3]

[16] 8. If $f(x) = \dfrac{4x-4}{x^2}$, then $f'(x) = 4\left[\dfrac{2-x}{x^3}\right]$ and $f''(x) = 8\left[\dfrac{x-3}{x^4}\right]$.

(a) Determine the intervals where $f(x)$ is increasing and where it is decreasing.
(b) Find all local maxima and all local minima.
(c) Determine where $f(x)$ is concave up and where it is concave down.
(d) Find all inflection points.
(e) Find all horizontal asymptotes and all vertical asymptotes.
(f) Sketch the graph of $f(x)$. [cf. Section 4.5]

Total = 70