MATH 1700 D01 Winter 2013 Assignment 4

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as e^2 as opposed to a decimal approximation.

1. Find the following limits using L'Hopital's Rule. Make sure to justify that you can use L'Hopital's Rule.
   (a) $\lim_{x \to 0} \frac{\cos 3x - 1}{2x}$
   (b) $\lim_{x \to 0} \frac{\cos 2x - 1 + 2x^2}{x^4}$
   (c) $\lim_{x \to -\infty} x \sin(1/x)$
   (d) $\lim_{x \to \infty} (xe^{2/x} - x)$
   (e) $\lim_{x \to 0^+} x^{\sqrt{x}}$
   (f) $\lim_{x \to 0} (1 + \sin x)^{2/x}$
2. Find whether the following improper integrals converge or diverge. If the integral converges, evaluate the integral.
   (a) $\int_0^\infty \frac{dz}{z^2 + 5z + 6}$
   (b) $\int_2^\infty \frac{x dx}{x^2 - 1}$
   (c) $\int_4^{12} \frac{2}{\sqrt[3]{x - 4}} dx$
3. Use the comparison test to determine whether the following improper integrals converge.
   (a) $\int_1^\infty \frac{e^{-x} + 3}{x^2} dx$
   (b) $\int_0^\infty \frac{1}{x^3 + \sqrt[3]{x}} dx$
4. Let R be the region below $y = \frac{1}{x^{2/3}}$, above the x-axis and to the right of $x = 1$. Note that this region is not bounded.
   (a) Show that the area of R is unbounded.
   (b) Show that the volume of R rotated about the x-axis is bounded and find the volume.

5. If $f(t)$ is continuous for $t \geq 0$, the Laplace Transform of f is the function F defined by

$$F(s) = \int_0^\infty f(t)e^{-st}dt.$$

Find the Laplace Transform of $f(t) = t^2$. Hint: Use can use without proof (although the proof would just require L'Hopital's Rule) that for all real numbers n,

$$\lim_{z\to\infty} z^n e^{-sz} = 0.$$