Given $f(x) = x^2 - 12x + 5$ on $[-1, 2]$.

(a) State the MVT

(b) verify the MVT for this function on the given interval.

Answer

(a) If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) then there exists at least one c where c is in (a,b) $(a < c < b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

(b) $f(x) = x^2 - 12x + 5$
on $[-1, 2]$.

$f'(x) = 2x - 12$

Clearly, $f(x)$ is continuous on $[-1, 2]$ because it is a polynomial.
Clearly, $f(x)$ is differentiable on $(-1, 2)$ because $f'(x) = 2x - 12$ exists for all x in $(-1, 2)$.
Therefore, the Mean Value Theorem (MVT) applies.

$f(x) = x^2 - 12x + 5$ on $[\underset{a}{-1}, \underset{b}{2}]$

$f(-1) = (-1)^2 - 12(-1) + 5 = 18$

$f(2) = (2)^2 - 12(2) + 5 = -15$

$$\frac{f(b) - f(a)}{b-a} = \frac{f(2) - f(-1)}{2-(-1)} = \frac{-15-18}{3} = \frac{-33}{3}$$

$$= \boxed{-11}$$

$$f'(x) = 2x - 12$$

$$f'(c) = 2c - 12$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$2c - 12 = -11$$

$$2c = -11 + 12$$

$$2c = 1$$

$$\boxed{c = \tfrac{1}{2} \text{ is in } (-1, 2)}$$

$\therefore$ We have verified MVT.