5. Verify that the following function satisfies the conditions of the Mean Value Theorem on the interval $[-3, 0]$, and then compute c which satisfies the conclusion of the Mean Value Theorem.

$$f(x) = \frac{x+6}{x+4}$$

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$$f'(x) = \frac{1(x+4) - (x+6)(1)}{(x+4)^2} = \frac{x+4-x-6}{(x+4)^2}$$

$$f'(x) = \frac{-2}{(x+4)^2}$$

Clearly, $f(x) = \frac{x+6}{x+4}$ is continuous on $[-3,0]$ because it's a rational function that is defined for all values in $[-3,0]$

Clearly, $f(x)$ is differentiable on $(-3,0)$ because $f'(x) = \frac{-2}{(x+4)^2}$ exists for all x on $(-3,0)$.

$\therefore$ Mean Value Theorem (MVT) applies.

$$f(x) = \frac{x+6}{x+4} \quad \text{on} \quad [\underset{a}{-3}, \underset{b}{0}]$$

$$f(0) = \frac{0+6}{0+4} = \frac{6}{4} = \frac{3}{2}$$

$$f(-3) = \frac{-3+6}{-3+4} = \frac{3}{1} = 3$$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(0)-f(-3)}{0-(-3)} = \frac{\frac{3}{2}-3}{3}$$

$$= \boxed{-\frac{1}{2}}$$

$$\frac{3}{2} - \frac{3}{1}$$

$$= \frac{3-6}{2}$$

$$= -\frac{3}{2}$$

$$\frac{-\frac{3}{2}}{3}$$

$$-\frac{3}{2} \div 3$$

$$-\frac{\cancel{3}}{2} \cdot \frac{1}{\cancel{3}}$$

$$= -\frac{1}{2}$$

$$f'(x) = \frac{-2}{(x+4)^2}$$

$$f'(c) = \boxed{\frac{-2}{(c+4)^2}}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\frac{-2}{(c+4)^2} = -\frac{1}{2}$$

Multiply both sides by -1

$$\frac{2}{(c+4)^2} = \frac{1}{2} \qquad \text{Cross-Multiply}$$

$$(c+4)^2 = 4$$

Square Root both sides

$c+4 = \pm\sqrt{4}$

$c+4 = 2$ or $c+4 = -2$

$c = -2$ or $\cancel{c = -6}$

because c must be in $(-3,0)$

$$\boxed{\begin{array}{l}c=-2 \text{ is in } (-3,0) \\ \text{and verifies the MVT}\end{array}}$$