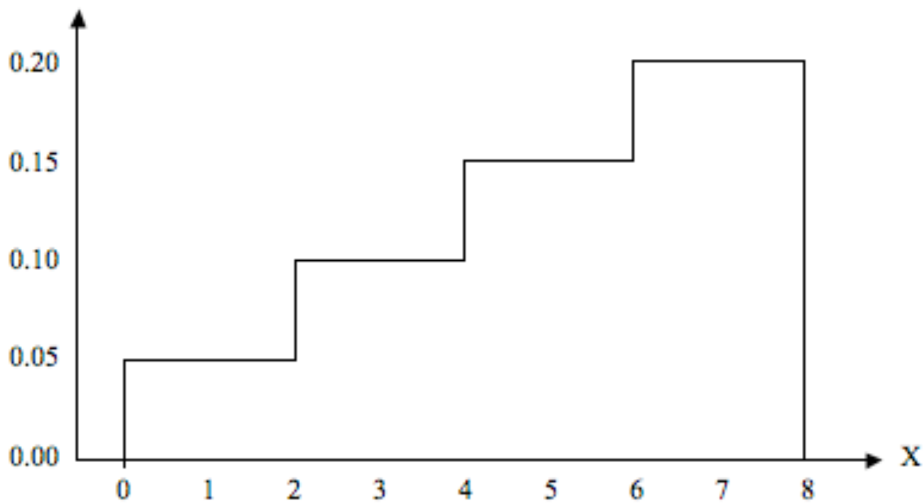
Sample Term Test 2 – A

1. A variable X has a distribution which is described by the density curve shown below:



What proportion of values of X fall between 1 and 6?

(A) 0.550 (B) 0.575 (C) 0.600 (D) 0.625 (E) 0.650

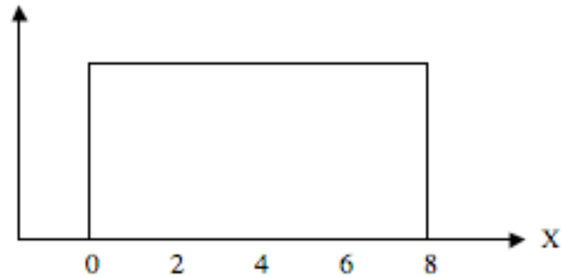
2. Which of the following statements about a normal distribution is true?

(A) The value of μ must always be positive.
(B) The value of σ must always be positive.
(C) The shape of a normal distribution depends on the value of μ.
(D) The possible values of a standard normal variable range from -3.49 to 3.49.
(E) The area under a normal curve depends on the value of σ.

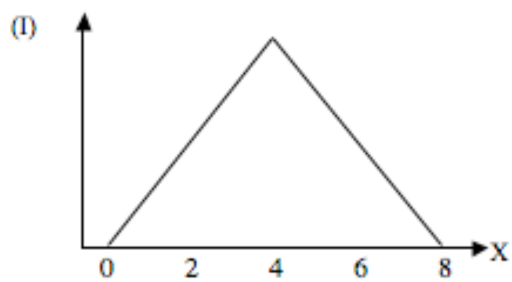
3. A variable X has a uniform distribution on the interval from 2 to 6. The $P(4.2 < X < 5.7)$ is equal to:

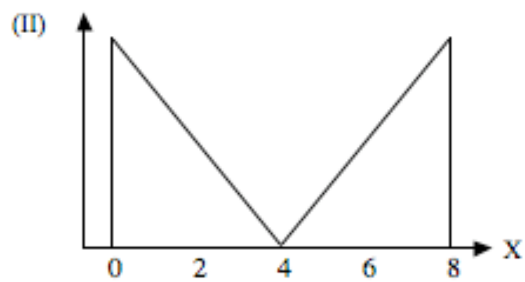
(A) 0.375 (B) 0.475 (C) 0.575 (D) 0.675 (E) 0.775

4. A variable X follows a uniform distribution, as shown below:



The distribution of X has an interquartile range equal to 4 (since the middle 50% of the data are contained between the values 2 and 6). Consider the variables with the distributions shown below (assume that the heights of the curves are such that they are both valid density curves):





The interquartile range of density curve (I) is ____________ and the interquartile range of density curve (II) is ____________.

(A) (I) less than 4, (II) greater than 4

(B) (I) greater than 4, (II) less than 4

(C) (I) equal to 4, (II) equal to 4

(D) (I) less than 4, (II) less than 4

(E) (I) greater than 4, (II) greater than 4

5. A variable Z has a standard normal distribution. What is the value b such that $P(-0.37 \leq Z \leq b) = 0.5749$?

(A) 2.02 (B) 1.48 (C) 0.97 (D) 0.63 (E) 1.72

The next **three** questions (**6** to **8**) refer to the following:

The sport of women's gymnastics consists of four events. Suppose it is known that scores for each event follow a normal distribution with the following means and standard deviations:

Event	Mean	Std. Dev.
Balance Beam	8.3	0.3
Uneven Bars	8.6	0.5
Vault	8.2	0.4
Floor Exercise	9.0	0.2

6. What proportion of gymnasts receives a score between 8.2 and 8.7 on the uneven bars?

(A) 0.2088 (B) 0.3674 (C) 0.6000 (D) 0.6837 (E) 0.3085

7. The top 6% of gymnasts in each event earn a trip to the national championships. What is the minimum vault score required to make it to the nationals?

(A) 8.44 (B) 7.58 (C) 8.90 (D) 8.41 (E) 8.82

8. Julie receives a score of 9.0 on the balance beam, 9.2 on the uneven bars, 9.1 on the vault and a 9.3 on floor exercise. In which event did Julie do the best relative to other gymnasts?

(A) Balance Beam
(B) Uneven Bars
(C) Vault
(D) Floor Exercise
(E) Julie did equally well on all events.

9. Suppose that the variable Z follows a standard normal distribution. If $P(-b < Z < b) = 0.92$, then b is approximately:

(A) 1.75 (B) 1.41 (C) 0.82 (D) 1.64 (E) 0.96

10. A variable X has a normal distribution with mean 100. It is known that about 47.5% of the values of X fall between 85 and 100. What is the approximate value of the standard deviation σ?

(A) 5 (B) 7.5 (C) 12.5 (D) 15 (E) 30

11. Speeds of vehicles on a highway follow a normal distribution with mean 106.2 km/h and standard deviation 8.7 km/h. What proportion of vehicles on this highway are travelling above the 100 km/h speed limit?

(A) 0.7126 (B) 0.2612 (C) 0.7910 (D) 0.2874 (E) 0.7611

12. The time to complete a particular exam is approximately normally distributed with a mean of 90 minutes and a standard deviation of 10 minutes. What percentage of students will take longer that 95 minutes to complete the exam?

(A) 50.00% (B) 69.15% (C) 30.85% (D) 19.15% (E) 22.85%

13. The time it takes skiers to finish a downhill race follows a normal distribution with mean 58.47 seconds and standard deviation 1.62 seconds. What proportion of skiers finish the race in exactly 60 seconds?

(A) 0.0556 (B) 0.1736 (C) 0.0122 (D) 0.0409 (E) 0.0000

14. Percentage grades in a large Calculus class follow a normal distribution with mean 60 and standard deviation 10. Percentage grades in a large English class follow a normal distribution with mean 68 and standard deviation 8. Katherine received a grade of 85 in Calculus. What grade does she need in English to be at the same percentile?

(A) 87 (B) 88 (C) 89 (D) 90 (E) 91

15. Bottles of a certain brand of apple juice are filled automatically by a machine. Fill volumes are normally distributed with a mean of μ and standard deviation of 1.46 ml. The label on the bottles claim that the bottles contain 500 ml of juice. What value should μ be set at so that only 2% of bottles will be underfilled?

    (A) 497 ml (B) 498 ml (C) 502 ml (D) 503 ml (E) 504 ml

16. A normal quantile plot is a useful tool to determine whether it is plausible that a variable has a normal distribution. Using a normal quantile plot, we conclude that a normal distribution is a reasonable assumption if:

    (A) all points fall close to a straight horizontal line.
    (B) all points fall close to a straight vertical line.
    (C) all points fall close to a straight diagonal line.
    (D) all points fall close to a bell-shaped curve.
    (E) the points appear randomly scattered.

17. It is known that 53% of students at a large university are female and 47% are male. If we take a random sample of 12 students at the university, what is the probability that exactly seven of them are female?

    (A) 0.1734 (B) 0.1834 (C) 0.1934 (D) 0.2034 (E) 0.2134

18. From past records, the professor of a large university course has established the following distribution for grades received by students in the course (with some values missing):

Grade	A+	A	B+	B	C+	C	D	F
Probability	0.08	0.17	???	0.13	???	0.22	0.09	0.07

    What is the probability of getting a grade of C+ or better?

    (A) 0.50
    (B) 0.62
    (C) 0.76
    (D) impossible to calculate without at least one of the missing probabilities.
    (E) impossible to calculate without both of the missing probabilities.

19. The number of courses X taken in one term by students at a large university has the probability distribution shown below, where k is some constant:

x	1	2	3	4	5	6
$P(X = x)$	0.05	k	0.21	$3k$	0.32	$2k$

What is the probability that a randomly selected student is taking at least 3 courses in one term?

(A) 0.81 (B) 0.53 (C) 0.76 (D) 0.88 (E) 0.69

The next **two** questions (**20** and **21**) refer to the following:

The following five games are scheduled to be played at the World Curling Championship one morning. The values in parentheses are the probabilities of each team winning their respective game.

Game 1: Finland (0.43)	vs.	Germany (0.57)
Game 2: USA (0.28)	vs.	Switzerland (0.72)
Game 3: Japan (0.11)	vs.	Canada (0.89)
Game 4: Denmark (0.33)	vs.	Sweden (0.67)
Game 5: France (0.18)	vs.	Scotland (0.82)

20. The outcome of interest is the set of winners of the five games. How many outcomes are contained in the sample space?

(A) 5 (B) 10 (C) 25 (D) 32 (E) 64

21. In a sports game, the "favourite" is the team with the higher probability of winning and the "underdog" is the team who is less likely to win. What is the probability that at least one underdog wins?

(A) 0.93 (B) 0.74 (C) 0.80 (D) 0.67 (E) 0.59

22. A manufacturer of automobile batteries claims that the distribution of battery lifetimes has a mean of 54 months and a variance of 36 months squared. Suppose a consumer group decides to check the claim by purchasing a sample of 50 of these batteries and subjecting them to tests to determine their lifetime. Assuming the manufacturer's claim is true, what is the probability that the sample has a mean lifetime less than 52 months?

(A) 0.1292 (B) 0.3707 (C) 0.0091 (D) 0.4909 (E) 0.3483

23. A recently married couple plans to have two children. The outcome of interest is the gender of each of the two children. Consider the event that exactly one of the couple's children will be a boy. Which of the following is the complement of this event?

(A) two boys
(B) two girls
(C) one girl
(D) at least one girl
(E) zero or two girls

24. The monthly mortgage payment for recent home buyers in Winnipeg has a mean of \$732, and a standard deviation of \$421. A random sample of 125 recent home buyers is selected. The approximate probability that their average monthly mortgage payment will be more than \$782 is:

(A) 0.9082 (B) 0.4522 (C) 0.4082 (D) 0.0478 (E) 0.0918

25. Weights of pears in an orchard follow a normal distribution with mean 195 grams and standard deviation 40 grams. A random sample of five pears is selected. What is the probability that the total weight of the pears is greater than one kilogram (i.e., 1,000 grams)?

(A) 0.2795 (B) 0.3897 (C) 0.2451 (D) 0.3264 (E) 0.4129

26. The fact that the sample mean does not tend to over- or underestimate the population mean makes the sample mean:

(A) resistant.
(B) unbiased.
(C) efficient.
(D) a statistic.
(E) a parameter.

27. According to the Canadian Blood Services website, 9% of Canadians have type B blood. If a sample of 8 donors is selected, what is the probability that less than two of them will have type B blood?

(A) 0.8424 (B) 0.9711 (C) 0.3721 (D) 0.4703 (E) 0.1576

28. A random variable X follows a uniform distribution with mean 3 and standard deviation 1.73. We take a random sample of size 100 from this distribution and calculate the sample mean $\bar{X}$. The sampling distribution of $\bar{X}$ is:

(A) approximately normal with mean 3 and standard deviation 0.173.

(B) uniform with mean 3 and standard deviation 1.73.

(C) approximately normal with mean 3 and standard deviation 0.0173.

(D) uniform with mean 3 and standard deviation 0.173.

(E) approximately normal with mean 3 and standard deviation 1.73.

29. There are four patients on the neo-natal ward of a local hospital who are monitored by two staff members. Suppose the probability (at any one time) of a patient requiring attention by a staff member is 0.3. Assuming the patients behave independently, what is the probability at any one time that there will not be sufficient staff to attend to all patients who need them?

(A) 0.0756 (B) 0.1104 (C) 0.0837 (D) 0.0463 (E) 0.2646

30. Which of the following variables has a binomial distribution?

(I) You repeatedly roll a fair die.
X = number of rolls needed to observe the number 6 for the third time.

(II) Tim Horton's is holding its annual "Roll Up the Rim to Win" promotion. Customers can check under the rim of a coffee cup to see if they have won a prize. You buy one cup of coffee from Tim Horton's each day for a week.
X = number of times you win a prize during the week.

(III) A quality control inspector in a factory routinely examines samples of fiber-optic cable being produced to check for defects. The inspector examines a 100-foot length of cable.
X = number of defects found on the cable.

(A) I only

(B) II only

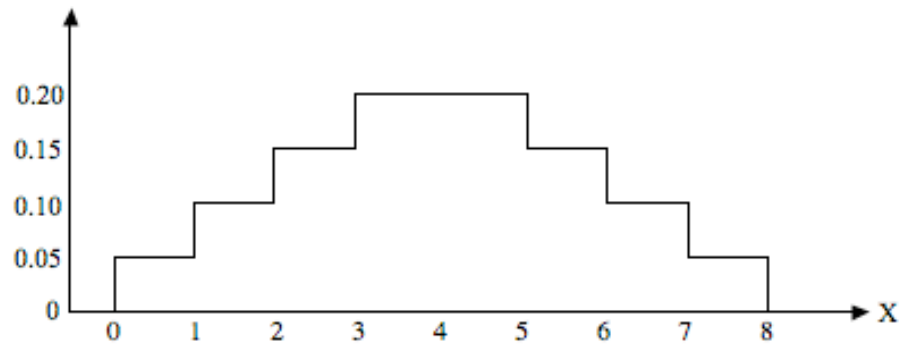
(C) I and II only

(D) II and III only

(E) I, II, and III

Sample Term Test 2 – B

1. A random variable X is described by the density curve shown below:



The probability of $P(3 \leq X \leq 6)$ is equal to:

(A) 0.55 (B) 0.45 (C) 0.375 (D) 0.40 (E) 0.60

2. Which of the following statements about a normal distribution is true?

(A) The mean of a normal distribution must always be greater than zero.
(B) For a standard normal distribution, $P(Z < z) = P(Z > -z)$ for any value z.
(C) The height of a normal density curve must always be equal to one.
(D) All values must fall within three standard deviations of the mean.
(E) The standard deviation of a normal distribution must always be greater than one.

3. Weights of apples grown in an orchard are known to follow a normal distribution with mean 160 grams. It is known that approximately 99.7% of apples have weights between 124 and 196 grams. What is the standard deviation of weights of all apples grown in the orchard?

(A) 9 grams (B) 12 grams (C) 18 grams (D) 24 grams (E) 36 grams

4. A variable Z has a standard normal distribution. What is the value b such that $P(b \leq Z \leq 0.36) = 0.2470$?

(A) -1.22 (B) -0.68 (C) -0.27 (D) -0.39 (E) -0.55

5. A variable X follows a normal distribution with mean 10 and standard deviation 5. Another variable Y follows a normal distribution with mean 25 and standard deviation 10. The maximum height of the density curve for X is _____(i)_____ the maximum height for the density curve for Y, and the area under the density curve for X is _____(ii)_____ the area under the density curve for Y.

   (A) (i) greater than, (ii) less than

   (B) (i) less than, (ii) greater than

   (C) (i) equal to, (ii) equal to

   (D) (i) greater than, (ii) equal to

   (E) (i) less than, (ii) less than

The next **two** questions (**6** and **7**) refer to the following:

Percentage grades in a large geography class follow a normal distribution with mean 67.5 and standard deviation 12.5.

6. What proportion of students in the class receive percentage grades between 60 and 70?

   (A) 0.2650 (B) 0.2750 (C) 0.2850 (D) 0.2950 (E) 0.3050

7. The professor decides to assign a grade of A+ to the students with the top 8% of the grades, and a grade of A to the next best 12%. What is the minimum percentage a student needs to earn a grade of A?

   (A) 77 (B) 78 (C) 79 (D) 80 (E) 81

8. The contents of bottles of water follow a normal distribution with mean μ and standard deviation 4 ml. What proportion of bottles have fill volumes within 1 ml of the mean?

   (A) 0.1974

   (B) 0.5987

   (C) 0.6826

   (D) 0.4013

   (E) unable to calculate without the value of μ

9. A candy company manufactures hard candies in five different flavours, according to the following probability distribution, where k is some constant:

Flavour	Cherry	Root Beer	Strawberry	Orange	Peppermint
Probability	0.21	k	0.27	k	0.14

If you select a candy at random, what is the probability that it will be fruit-flavoured?

(A) 0.48 (B) 0.67 (C) 0.68 (D) 0.72 (E) 0.86

10. Event A has probability of 0.4 to occur and Event B has a probability of 0.5 to occur. Their union (A or B) has a probability of 0.7 to occur. Then:

(A) A and B are mutually exclusive.

(B) A and B are not mutually exclusive.

(C) A and B are independent.

(D) A and B are dependent.

(E) both (B) and (C).

11. You have two unfair coins. On any given flip, the first coin has a 40% chance of landing on Heads and the second coin has a 25% chance of landing on Heads. If you flip both coins, what is the probability that at least one of them lands on Heads?

(A) 0.10 (B) 0.45 (C) 0.55 (D) 0.65 (E) 0.90

12. Suppose it is known that 8% of males are colour blind. In a random sample of 20 males, what is the probability that exactly 3 of them are colour blind?

(A) 0.1212 (B) 0.1313 (C) 0.1414 (D) 0.1515 (E) 0.1616

13. Suppose it is known that 61% of Winnipeg adults read the Winnipeg Free Press, 29% read the Winnipeg Sun and 18% read both newspapers. What is the probability that a randomly selected adult in Winnipeg reads only one of the two papers (but not both)? Hint: Draw a Venn diagram.

(A) 0.54 (B) 0.50 (C) 0.45 (D) 0.72 (E) 0.47

The next **two** questions (**14** and **15**) refer to the following:

A hockey players compiles the following facts:

- Her team wins (W) 60% of their games.
- She scores a goal (G) in 30% of her games.
- She gets a penalty (P) in 40% of her games.
- In 38% of her games, her team wins and she scores a goal.
- In 24% of her games, her team wins and she gets a penalty.
- in 15% of her games, she scores a goal and gets a penalty.

14. In any given game, what is the probability that the player scores a goal or gets a penalty?

    (A) 0.85 (B) 0.55 (C) 0.70 (D) 0.65 (E) 0.58

15. Which of the following statements is **true**?

    (A) W and G are independent.
    (B) G and P are mutually exclusive (disjoint).
    (C) W and P are independent.
    (D) W and G are mutually exclusive (disjoint).
    (E) G and P are independent.

16. Which of the following variables has a binomial distribution?

    (A) You roll five fair dice, each with face values of 1 through 6.
    X = total number of dots facing up on the five dice.
    (B) An airplane carrying 100 passengers has two emergency exits, one at the front of the plane and one at the back. The airplane makes an emergency landing.
    X = number of people who leave the plane through the front exit.
    (C) A paper boy delivers the newspaper to every house on your block.
    X = number of houses that get their newspaper on time tomorrow morning.
    (D) You repeatedly flip two quarters simultaneously until both quarters land on Heads.
    X = number of flips required for both quarters to land on Heads.
    (E) A student randomly guesses the answer to each of the 40 multiple choice questions on this exam.
    X = number of multiple choice questions the student gets correct.

17. The probability that a certain machine will produce a defective item is $\frac{1}{4}$. If a random sample of six items is taken from the output of this machine, what is the probability that there will be at least five defectives in the sample?

(A) $\dfrac{1}{4096}$ (B) $\dfrac{3}{4096}$ (C) $\dfrac{4}{4096}$ (D) $\dfrac{18}{4096}$ (E) $\dfrac{19}{4096}$

18. A random variable X follows a binomial distribution with parameters n and p. If the mean and the variance of X are 3.6 and 2.52 respectively, then the values of the parameters n and p are, respectively:

(A) 6 and 0.6.
(B) 18 and 0.2.
(C) 24 and 0.15.
(D) 12 and 0.3.
(E) 12 and 0.4.

19. A student driving to university must pass through seven sets of traffic lights. Suppose it is known that each set of traffic lights are red 35% of the time and that all lights function independently. What is the probability that the student will have to stop at two or more sets of lights on her to university?

(A) 0.6828 (B) 0.2985 (C) 0.4893 (D) 0.7662 (E) 0.5997

The next **three** questions (**20** to **22**) refer to the following:

> We have a small deck of ten cards. Five of the cards are red, three are blue and two are green. We randomly select **four** cards from the deck **with replacement**. That is, after we select a card and record the colour, we put the card back in the deck and thoroughly shuffle them before we select another card.

20. Let X be the number of blue cards that are selected. The distribution of X is:

    (A) binomial with parameters $n = 4$ and $p = 0.1$.

    (B) binomial with parameters $n = 10$ and $p = 0.3$.

    (C) binomial with parameters $n = 10$ and $p = 0.4$.

    (D) binomial with parameters $n = 4$ and $p = 0.3$.

    (E) normal with parameters $\mu = 1.2$ and $\sigma = 0.92$.

21. Let A be the event that the first selected card is the only red card in our four selections. Which of the following events is mutually exclusive (disjoint) from the event A?

    (A) Second card selected is blue.

    (B) No green cards are selected.

    (C) Third selected card is the only green.

    (D) Same number of red and blue cards are selected.

    (E) Same number of blue and green cards are selected.

22. What is the probability that the first two selected cards are the same colour?

    (A) 0.38 (B) 0.25 (C) 0.29 (D) 0.33 (E) 0.41

The next **two** questions (**23** and **24**) refer to the following:

> The time X taken by a cashier in a grocery store express lane follows a normal distribution with mean 90 seconds and standard deviation 20 seconds.

23. What is the first quartile (Q_1) of the distribution of X?

    (A) 73.8 seconds
    (B) 85.0 seconds
    (C) 69.4 seconds
    (D) 81.2 seconds
    (E) 76.6 seconds

24. What is the probability that the average service time for the next three customers is between 80 and 100 seconds? (Assume the next three customers can be considered a simple random sample.)

    (A) 0.6156
    (B) 0.4893
    (C) 0.7212
    (D) 0.5559
    (E) impossible to calculate with the information given

25. Weights of oranges sold at a supermarket follow a normal distribution with mean 0.22 pounds and standard deviation 0.04 pounds. If you randomly select four oranges, what is the probability that their total weight is less than 1 pound?

    (A) 0.9719 (B) 0.8508 (C) 0.9332 (D) 0.7967 (E) 0.8340

The next **two** questions (**26** and **27**) refer to the following:

The amount X spent (in \$) by customers in the grocery store express lane follow some right-skewed distribution with mean \$24 and standard deviation \$15.

26. What is the probability that the average amount spent by the next three customers is more than \$20? (Assume the next three customers can be considered a simple random sample.)

    (A) 0.4619
    (B) 0.6772
    (C) 0.8186
    (D) 0.7673
    (E) impossible to calculate with the information given

27. What is the probability that the next 40 customers spend less than \$1,000 in total? (Assume the next 40 customers can be considered a simple random sample).

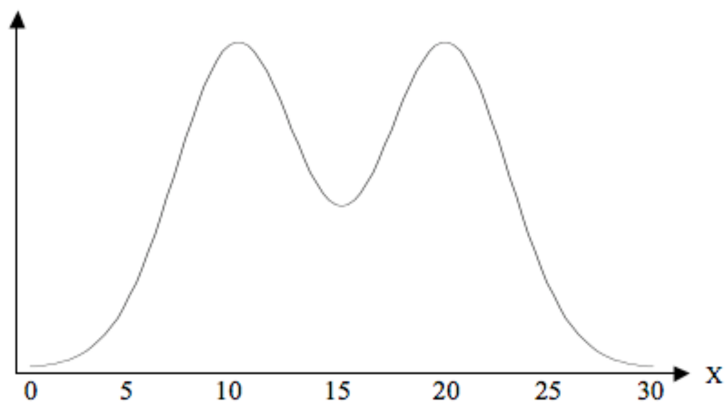
    (A) 0.5199 (B) 0.6064 (C) 0.6628 (D) 0.5784 (E) 0.6331

28. The Central Limit Theorem states that:

    (A) when n gets large, the standard deviation of the sample mean $\bar{X}$ gets closer and closer to $\frac{\sigma}{\sqrt{n}}$.
    (B) regardless of the population distribution of a random variable X, when n gets large, the sampling distribution of $\bar{X}$ is approximately normal.
    (C) if a random variable X follows a normal distribution, then when n gets large, the sampling distribution of $\bar{X}$ is exactly normal.
    (D) when n gets large, the sample mean $\bar{X}$ gets closer and closer to the population mean μ.
    (E) when n gets large, the sample mean $\bar{X}$ becomes an unbiased estimator of the population mean μ.

The next **two** questions (**29** and **30**) refer to the following:

A bimodal probability distribution is one with two distinct peaks. A random variable X follows a bimodal distribution with mean 15 and standard deviation 4, as shown below:



29. Suppose that you take a random sample of 10,000 observations from the population above and make a histogram. You expect the histogram to be:

(A) approximately normal with mean close to 15 and standard deviation close to 0.0004.
(B) bimodal with mean close to 15 and standard deviation close to 0.04.
(C) approximately normal with mean close to 15 and standard deviation close to 0.04.
(D) bimodal with mean close to 15 and standard deviation close to 4.
(E) approximately normal with mean close to 15 and standard deviation close to 4.

30. Suppose that you take 10,000 random samples of 10,000 observations from the population above and that for each sample, the mean $\bar{x}$ is calculated. A histogram of resulting $\bar{x}$'s would be:

(A) approximately normal with mean close to 15 and standard deviation close to 0.0004.
(B) bimodal with mean close to 15 and standard deviation close to 0.04.
(C) approximately normal with mean close to 15 and standard deviation close to 0.04.
(D) bimodal with mean close to 15 and standard deviation close to 4.
(E) approximately normal with mean close to 15 and standard deviation close to 4.

Sample Term Test 2 – Solutions

Question	Sample Test A	Sample Test B
1	A	A
2	B	B
3	A	B
4	A	C
5	B	D
6	B	E
7	E	B
8	A	A
9	A	B
10	B	E
11	E	C
12	C	C
13	E	A
14	B	B
15	D	C
16	C	E
17	E	E
18	B	D
19	D	D
20	D	D
21	C	E
22	C	A
23	E	E
24	E	A
25	B	C
26	B	E
27	A	C
28	A	B
29	C	D
30	B	C