Sample Midterm 1

1. We calculate that, in order to estimate the true mean annual salary of all workers in a large union to within \$400 with 95% confidence, we need a sample of 500 workers. What sample size would be required to estimate the true mean salary to within \$500 with 95% confidence?

   (A) 320 (B) 400 (C) 448 (D) 625 (E) 782

2. The sizes of farms in a U.S. state follow a normal distribution with standard deviation 30 acres. Suppose we measure the size of a random sample of farms and calculate the 98% confidence interval to be (295, 305). What is the correct interpretation of this interval?

   (A) Approximately 98% of farms have a size between 295 and 305 acres.

   (B) Approximately 98% of samples of 30 farms will have a mean size between 295 and 305 acres.

   (C) The probability that the population mean is between 295 and 305 acres is 98%.

   (D) In repeated samples of the same size, 98% of similarly constructed intervals will contain the sample mean.

   (E) In repeated samples of the same size, 98% of similarly constructed intervals will contain the population mean.

3. We would like to conduct a hypothesis test to examine whether there is evidence that the true mean amount spent on textbooks by a U of M student in one semester differs from \$400. A random sample of 50 students is selected and the mean amount they spent on textbooks for one semester is calculated to be \$430. Assume the population standard deviation is known to be \$165. What is the P-value for the appropriate hypothesis test?

   (A) 0.1970 (B) 0.1112 (C) 0.0630 (D) 0.0985 (E) 0.1260

4. We take a simple random sample of 16 adults and ask them how long they sleep on a typical night. The sample mean is calculated to be 7.2 hours. Suppose it is known that the number of hours adults sleep at night follows a normal distribution with standard deviation 1.8 hours. An 88% confidence interval for the true mean time adults sleep at night is:

   (A) (6.80, 7.60)
   (B) (6.30, 8.10)
   (C) (6.60, 7.80)
   (D) (6.70, 7.70)
   (E) (6.50, 7.90)

5. Which of the following statements comparing the standard normal distribution and the t distributions is **false**?

   (A) The density curve for Z is taller at the center than the density curve for T.
   (B) The t distributions have more area in the tails than the standard normal distribution.
   (C) In tests of significance for μ, Z should be used as the test statistic when the distribution of X is normal, and T should be used in other cases.
   (D) As the sample size increases, the t distribution approaches the standard normal distribution.
   (E) In tests of significance for μ, T should be used as the test statistic only when the population standard deviation is unknown.

6. The Manitoba Department of Agriculture would like to estimate the average yield per acre of a new variety of corn for farms in southwestern Manitoba. It is desired that the final estimate be within five bushels per acre of the true mean yield. Due to cost restraints, a sample of no more than 65 one-acre plots of land can be obtained to conduct an experiment. The population standard deviation is known to be 17.33 bushels per acre. What is (approximately) the maximum confidence level that could be attained for a confidence interval that meets the Agriculture Department's specification?

   (A) 90% (B) 95% (C) 96% (D) 98% (E) 99%

7. A statistical test of significance is designed to:

   (A) prove that the null hypothesis is true.
   (B) prove that the alternative hypothesis is true.
   (C) find the probability that the null hypothesis is true.
   (D) assess the strength of evidence in favour of the null hypothesis.
   (E) assess the strength of evidence in favour of the alternative hypothesis.

8. We take a random sample of 12 observations from a normally distributed population. These 12 observations have a mean of 71.3 and standard deviation of 5.9. A confidence interval for μ is calculated to be **(67.335, 75.265)**. The confidence level of this interval is closest to:

   (A) 90% (B) 95% (C) 96% (D) 98% (E) 99%

The next **two** questions (**9** and **10**) refer to the following:

The systems department of the Texaco Oil Company runs a large number of simulation programs on their mainframe computer. A manufacturer of new simulation software claims their program will run the simulation faster than the current mean of 30 minutes. The new program will be run 20 times. A hypothesis test of $\mathrm{H}_0 : \mu = 30$ vs. $\mathrm{H}_\mathrm{a} : \mu < 30$ is to be conducted, and it is decided that H_0 will be rejected if $\bar{x} \leq 28.3$ minutes. Run times for the new program are known to follow a normal distribution with standard deviation 3.7 minutes.

9. The level of significance of the test is closest to:

   (A) 0.01 (B) 0.02 (C) 0.03 (D) 0.04 (E) 0.05

10. What is the probability of making a Type II error if the true mean run time for the new program is actually 27 minutes?

   (A) 0.0192 (B) 0.0244 (C) 0.0384 (D) 0.0475 (E) 0.0582

The next **two** questions (**11** and **12**) refer to the following:

> The weights of apples in a large orchard are known to follow a normal distribution with a standard deviation of 12.2 grams. A random sample of 15 apples is selected from the orchard. We would like to conduct a hypothesis test at the 5% level of significance to determine whether the true mean weight of all apples in the orchard is greater than 150 grams.

11. What is the power of the hypothesis test if the true mean weight of all apples in the orchard is actually 160 grams?

    (A) 0.9115 (B) 0.9370 (C) 0.9500 (D) 0.9726 (E) 0.9918

12. In the previous question, all else remaining the same, which of the following would have resulted in a higher power?

    (I) selecting a random sample of 25 apples
    (II) if the true mean weight of all apples was actually 170 grams
    (III) using a 10% level of significance

    (A) I only
    (B) I and II only
    (C) I and III only
    (D) II and III only
    (E) I, II and III

13. A statistician conducted a test of $\mathrm{H_0}: \mu = 100$ vs. $\mathrm{H_a}: \mu \neq 100$ for the mean μ of some normally distributed population. Based on the gathered data, the statistician calculated a sample mean of $\bar{x} = 110$ and concluded that $\mathrm{H_0}$ could be rejected at the 5% level of significance. Using the same data, which of the following statements **must** be true?

(I) A test of $\mathrm{H_0}: \mu = 100$ vs. $\mathrm{H_a}: \mu > 100$ at the 5% level of significance would also lead to rejecting $\mathrm{H_0}$.

(II) A test of $\mathrm{H_0}: \mu = 90$ vs. $\mathrm{H_a}: \mu \neq 90$ at the 5% level of significance would also lead to rejecting $\mathrm{H_0}$.

(III) A test of $\mathrm{H_0}: \mu = 100$ vs. $\mathrm{H_a}: \mu \neq 100$ at the 1% level of significance would also lead to rejecting $\mathrm{H_0}$.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II and III

14. We take random samples of individuals from two normally distributed populations and record the value of some random varialbe X. Some summary statistics are shown below:

Population	Sample Size	Sample Mean	Sample Std. Dev.
1	7	29	4.0
2	14	25	9.2

We would like to contstruct a 95% confidence interval for the difference $\mu_1 - \mu_2$ in population means. What is the value of the standard error of $\bar{X}_1 - \bar{X}_2$?

(A) 2.24 (B) 1.86 (C) 3.27 (D) 2.89 (E) 3.45

The next **three** questions (**15**, **16** and **17**) refer to the following:

Coke and Pepsi are the two most popular brands of cola on the market. Do consumers prefer either one of the two brands of cola over the other? We conduct an experiment as follows: 20 volunteers participate in a blind taste test. Each volunteer tastes both Coke and Pepsi (in random order) and scores the taste of each cola on a scale from 0 to 100. Some information that may be helpful is shown in the table below:

Scores for Coke	Scores for Pepsi	Difference (Coke − Pepsi)
mean = 78	mean = 83	mean = −5
std. dev. = 27	std. dev. = 24	std. dev. = 13

15. Which of the following statements is/are **true**?

(I) The scores for Coke and Pepsi for each individual are independent.
(II) The scores for Coke and Pepsi for each individual are dependent.
(III) In order to conduct the matched pairs t test, we must assume that scores for Coke and scores for Pepsi both follow normal distributions.
(IV) In order to conduct the matched pairs t test, we must assume that the difference in scores (Coke − Pepsi) follow a normal distribution.

(A) I only
(B) I and III only
(C) I and IV only
(D) II and III only
(E) II and IV only

16. We will use the critical value method to conduct the test, with a 10% level of significance. Assuming the appropriate assumptions are satisfied, the rejection rule is to reject H_0 if:

(A) $|t| \geq 1.328$ (B) $t \geq 1.645$ (C) $|t| \geq 1.729$ (D) $t \geq 1.282$ (E) $|t| \geq 1.833$

17. Assuming the appropriate assumptions are satisfied, what is the value of the test statistic for the appropriate test of significance?

(A) -0.38 (B) -7.69 (C) -2.85 (D) -1.72 (E) -0.88

18. We record the amount of time (in hours) spent watching TV per week for random samples of young children and teenagers. Some summary statistics are shown below:

	Sample Size	Mean	Standard Deviation
Children	16	26.4	7.3
Teenagers	9	19.5	6.2

Weekly TV viewing times follow normal distributions for both children and teenagers. An appropriate hypothesis test is conducted at the 1% level of significance to determine whether there is a difference in the true mean TV viewing time for children and teenagers. The test statistic is calculated to be $t = 2.39$. We conclude that we should:

(A) reject H_0, since the P-value is between 0.01 and 0.02.

(B) fail to reject H_0, since the P-value is between 0.01 and 0.02.

(C) reject H_0, since the P-value is between 0.02 and 0.04.

(D) fail to reject H_0, since the P-value is between 0.02 and 0.04.

(E) reject H_0, since the P-value is between 0.04 and 0.05.

19. Random samples of contestants who have appeared on two popular game shows (*Wheel of Fortune* and *Jeopardy*) are selected. The Wheel of Fortune contestants in the sample won an average of $5700 more than the Jeopardy contestants in the sample. A 95% confidence interval for the difference in the mean amount of money won by contestants on the two game shows is calculated to be $(-1900, 13300)$. We would like to conduct a hypothesis test to determine whether the mean amounts won on the two game shows differ. Assume the appropriate normality assumptions are satisfied. We would:

(A) reject H_0 at the 5% level of significance since the value 0 is contained in the 95% confidence interval.

(B) fail to reject H_0 at the 5% level of significance since the value 0 is contained in the 95% confidence interval.

(C) fail to reject H_0 at the 5% level of significance since the value 5700 is contained in the 95% confidence interval.

(D) reject H_0 at the 10% level of significance since the value 5700 is contained in the 95% confidence interval.

(E) fail to reject H_0 at the 10% level of significance since the value 0 is contained in the 95% confidence interval.

The next **two** questions (**20** and **21**) refer to the following:

Several workers in a large office building suspect that coffee vending machine A dispenses more coffee on average than vending machine B. To test this belief, several samples were taken from each machine. The amounts (in ounces) dispensed and some summary statistics are given below:

Machine	n	$\bar{x}$	s
A	12	11.74	0.56
B	8	11.18	0.79

20. Assuming fill volumes follow a normal distribution for both machines, the test statistic for the appropriate test of significance is:

(A) $$\frac{11.74 - 11.18}{\sqrt{0.46\left(\frac{1}{12}+\frac{1}{8}\right)}}$$

(B) $$\frac{11.74 - 11.18}{\sqrt{\frac{(0.56)^2}{12}+\frac{(0.79)^2}{8}}}$$

(C) $$\frac{11.74 - 11.18}{\sqrt{0.43\left(\frac{1}{12}+\frac{1}{8}\right)}}$$

(D) $$\frac{11.74 - 11.18}{\sqrt{\frac{(0.56)^2}{11}+\frac{(0.79)^2}{7}}}$$

(E) $$\frac{11.74 - 11.18}{\sqrt{0.68\left(\frac{1}{12}+\frac{1}{8}\right)}}$$

21. We would make a Type II error in this test if we concluded that:

(A) Machine A dispenses more on average than Machine B when in fact Machine B dispenses more on average than Machine A.

(B) there is no evidence that Machine A and B dispense the same amount on average when in fact Machine A does dispense more on average.

(C) Machine B dispenses more on average than Machine A when in fact Machine A dispenses more on average than Machine B.

(D) Machine A dispenses more on average than Machine B when in fact Machines A and B dispense the same amount on average.

(E) there is no evidence that Machine A dispenses more on average than Machine B when in fact Machine A does dispense more on average.

22. We would like to conduct a hypothesis test at the 5% level of significance to determine whether hockey players weigh more on average than baseball players. We record the weights of a random sample of 12 professional hockey players and 7 professional baseball players. Weights of athletes in both sports are known to follow normal distributions. Some *JMP* output that may be helpful is shown below:

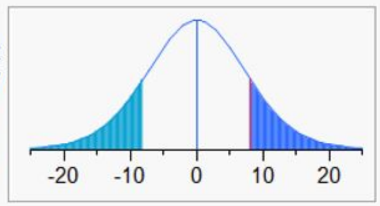
Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
Baseball	7	187.571	7.7213	2.9184	180.43	194.71
Hockey	12	195.583	18.3920	5.3093	183.90	207.27

t Test

Hockey-Baseball
Assuming equal variances

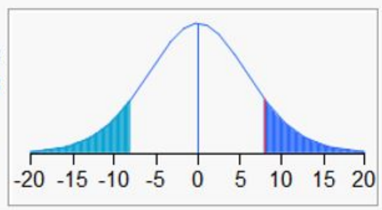
Difference	8.012	t Ratio	1.087592
Std Err Dif	7.367	DF	17
Upper CL Dif	23.554	Prob > |t|	0.2920
Lower CL Dif	-7.530	Prob > t	0.1460
Confidence	0.95	Prob < t	0.8540



t Test

Hockey-Baseball
Assuming unequal variances

Difference	8.012	t Ratio	1.322418
Std Err Dif	6.059	DF	15.97728
Upper CL Dif	20.857	Prob > |t|	0.2046
Lower CL Dif	-4.833	Prob > t	0.1023
Confidence	0.95	Prob < t	0.8977



Based on the data, we would fail to reject the null hypothesis of the appropriate test of significance, since

(A) $t = 1.32 < t^* = 1.753$

(B) $t = 1.09 < t^* = 1.740$

(C) $t = 1.32 < t^* = 1.645$

(D) $t = 1.09 < t^* = 2.110$

(E) $t = 1.32 < t^* = 2.131$

23. We would like to conduct a hypothesis test to determine whether females (F) can complete a certain task faster than males (M) on average. We will commit a Type I Error if we conclude that:

(A) $\mu_F > \mu_M$ when in fact $\mu_F = \mu_M$

(B) $\mu_F < \mu_M$ when in fact $\mu_F > \mu_M$

(C) $\mu_F = \mu_M$ when in fact $\mu_F < \mu_M$

(D) $\mu_F < \mu_M$ when in fact $\mu_F = \mu_M$

(E) $\mu_F > \mu_M$ when in fact $\mu_F < \mu_M$

The next **two** questions (**24** and **25**) refer to the following:

We conduct an experiment to compare the responses to six different treatments. Volunteers are randomly assigned to receive one of the six treatments. The response variable is measured for each individual and an analysis of variance F test is conducted to assess the equality of the means for the six treatments. Responses for each of the six treatments are known to follow normal distributions. Some summary statistics are shown below:

Treatment	Sample Size	Sample Mean	Sample Std. Dev.
1	3	17	8
2	5	14	7
3	2	21	11
4	4	20	6
5	3	18	9
6	4	12	10

24. Under the null hypothesis, we assume all population means are equal. What is the estimate of this common population mean?

(A) 15.99 (B) 16.43 (C) 16.78 (D) 17.00 (E) 17.22

25. What is the 5% critical value for the appropriate test of significance?

(A) 2.57 (B) 2.68 (C) 2.79 (D) 2.85 (E) 2.90

The next **three** questions (**26** to **28**) refer to the following:

Battery life of MP3 players is of great concern to customers. A consumer group tested four brands of MP3 players to determine the battery life. Samples of players of each brand were fully charged and left to run on medium volume until the battery died. The following table displays the number of hours each of the batteries lasted:

Brand

	A	B	C	D
	24	26	18	27
	19	28	20	24
	22	24	21	22
		25		24
mean	21.67	25.75	19.67	24.25
std. dev.	2.52	1.71	1.53	2.06

We would like to conduct an analysis of variance to compare the performance of the four brands. Suppose that all necessary assumptions have been satisfied. The ANOVA table (with some values missing) is shown below:

Source of Variation	*df*	Sum of Squares	Mean Squares	F
Groups				
Error			3.88	
Total		113.71		

26. What is the alternative hypothesis for the appropriate test of significance?

(A) H_a: All population variances are different.

(B) H_a: At least one sample mean differs.

(C) H_a: All population means are different.

(D) H_a: At least one population variance differs.

(E) H_a: At least one population mean differs.

27. What is the P-value of the appropriate test of significance?

(A) less than 0.001
(B) between 0.01 and 0.025
(C) between 0.025 and 0.05
(D) between 0.05 and 0.10
(E) greater than 0.10

28. We would like to construct a 95% confidence interval for the true mean battery life for all Brand B MP3 players. The 95% confidence interval is:

(A) (23.56, 27.94)
(B) (23.17, 28.33)
(C) (23.03, 28.47)
(D) (22.51, 28.99)
(E) (22.04, 29.46)

The next **two** questions (**29** and **30**) refer to the following:

We conduct an experiment to compare the responses to five different treatments. Volunteers are randomly assigned to receive one of the five treatments. Each treatment is assigned to the same number of individuals. The response variable is measured for each individual and an analysis of variance F test is conducted to assess the equality of the means for the five treatments. Responses for each of the five treatments are known to follow normal distributions with common standard deviation. The ANOVA table (with some values missing) is shown below:

Source of Variation	*df*	Sum of Squares	Mean Squares	F
Groups		74.52		
Error	60		5.78	
Total				

29. How many individuals received each of the five treatments?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

30. One assumption required in conducting and ANOVA F test is that all population standard deviations are equal. The estimate of this common population standard deviation is:

(A) 2.40 (B) 4.32 (C) 5.78 (D) 18.63 (E) 33.41

Answer Key

1. A
2. E
3. A
4. E
5. C

6. D
7. E
8. C
9. B
10. E

11. B
12. E
13. C
14. D
15. E

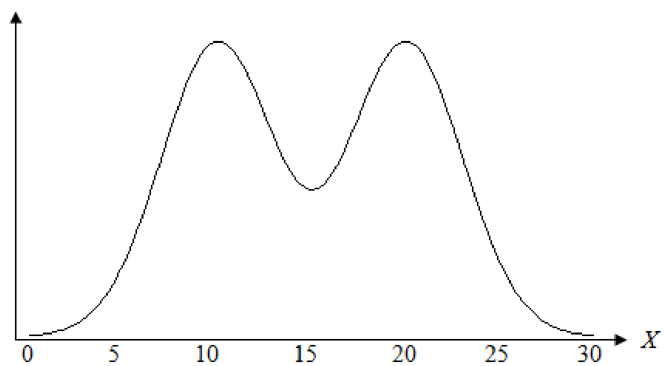
16. C
17. D
18. D
19. B
20. C

21. E
22. A
23. D
24. B
25. E

26. E
27. B
28. A
29. C
30. A

Sample Midterm 2

1. A bimodal probability distribution is one with two distinct peaks. A random variable X follows a bimodal distribution with mean 15 and standard deviation 4, as shown below:



We will take a random sample of 10,000 individuals from this distribution and calculate the sample mean $\bar{x}$. The sampling distribution of $\bar{X}$ is:

(A) approximately normal with mean close to 15 and standard deviation close to 0.0004.

(B) bimodal with mean close to 15 and standard deviation close to 0.04.

(C) approximately normal with mean close to 15 and standard deviation close to 0.04.

(D) bimodal with mean close to 15 and standard deviation close to 4.

(E) approximately normal with mean close to 15 and standard deviation close to 4.

2. GPAs at a large university follow a normal distribution with mean 2.84 and standard deviation 0.46. What is the probability that a random sample of four students has a mean GPA greater than 3.00?

(A) 0.2420

(B) 0.6957

(C) 0.7580

(D) 0.3043

(E) impossible to calculate with the information given.

3. Annual salaries of workers in a large union follow a normal distribution with standard deviation \$10,000. What sample size is required if we want to estimate the true mean salary to within \$2,000 with 93% confidence?

(A) 82 (B) 85 (C) 88 (D) 94 (E) 97

4. We will take a random sample of 30 vehicles of a certain make and model and measure the fuel efficiency in miles per gallon (mpg) of each of them. We will conduct a hypothesis test at the 10% level of significance to determine whether there is evidence that the true mean fuel efficiency of all cars of this make and model differs from 32 mpg. What is the probability of failing to reject H_0 if the true mean is in fact 32 mpg?

(A) 0.10 (B) 0.95 (C) 0.05 (D) 0.90 (E) 0.20

5. For which of the following tests would a Type II Error have more serious consequences than a Type I Error?

(I) A police officer is trying to decide whether to pull over a driver.
H_0: The driver is not drunk.
H_a: The driver is drunk.

(II) You are deciding whether to go for a swim in the ocean.
H_0: There are no sharks in the water.
H_a: There are sharks in the water.

(III) A shoplifter is deciding whether to steal an expensive camera from Walmart.
H_0: He will not be caught.
H_a: He will be caught.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III

6. A horticulturist wishes to estimate the true mean growth of seedlings in a large timber plot last year. A random sample of 20 seedlings is selected and the one-year growth for each of them is measured. The sampled seedlings have a mean growth of 5.6 cm and a standard deviation of 1.5 cm. One-year seedling growth is known to follow a normal distribution with standard deviation 2.0 cm. We wish to conduct a hypothesis test at the 10% level of significance to determine whether the true mean one-year seedling growth differs from 6.0 cm. The critical region for the test is:

(A) $|z| > 1.645$

(B) $t < -1.328$

(C) $z < -1.282$

(D) $|t| > 1.729$

(E) $|z| > 1.282$

7. A random variable X follows a normal distribution with standard deviation $\sigma = 8$. We take a random sample of 25 individuals from the population and we calculate a confidence interval for μ to be **(36.7136, 43.2864)**. What is the confidence level of this interval?

(A) 90% (B) 95% (C) 96% (D) 98% (E) 99%

8. A test of $H_0 : \mu = 100$ vs. $H_a : \mu \neq 100$ is conducted for the mean μ of some population. We take a sample of ten individuals and calculate a sample mean of 104. A 98% confidence interval for μ is calculated to be (101, 107). Which of the following statements is **true**?

(A) We fail to reject H_0 at a 4% level of significance, since 100 is not contained in the 98% confidence interval.

(B) We fail to reject H_0 at a 2% level of significance, since 104 is contained within the 98% confidence interval.

(C) We reject H_0 at the 1% level of significance, since 100 is not contained within the 98% confidence interval.

(D) We fail to reject H_0 at the 4% level of significance, since 104 is contained within the 98% confidence interval.

(E) We reject H_0 at a 2% level of significance, since 100 is not contained within the 98% confidence interval.

9. A simple random sample of six male patients over the age of 65 is being used in a blood pressure study. The standard error of the mean blood pressure of these six men was calculated to be 7.8. What is the standard deviation of these six blood pressure measurements?

(A) 2.8 (B) 3.2 (C) 14.4 (D) 19.1 (E) 46.8

10. A statistician conducted a test of $\mathrm{H_0} : \mu = 1$ vs. $\mathrm{H_a} : \mu > 1$ for the mean μ of some population. Based on the gathered data, the statistician concluded that $\mathrm{H_0}$ could be rejected at the 1% level of significance. Using the same data, which of the following statements must be true?

(I) A test of $\mathrm{H_0} : \mu = 1$ vs. $\mathrm{H_a} : \mu > 1$ at the 10% level of significance would also lead to rejecting $\mathrm{H_0}$.
(II) A test of $\mathrm{H_0} : \mu = 0$ vs. $\mathrm{H_a} : \mu > 0$ at the 1% level of significance would also lead to rejecting $\mathrm{H_0}$.
(III) A test of $\mathrm{H_0} : \mu = 1$ vs. $\mathrm{H_a} : \mu \neq 1$ at the 1% level of significance would also lead to rejecting $\mathrm{H_0}$.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III

11. The Winnipeg Transit Commission claims that the average time taken by the Number 60 bus to travel from the University of Manitoba to downtown is 27 minutes. A student who takes this route often believes that the true mean time is greater than 27 minutes. The student records the time for a sample of five trips. These trips had a mean time of 30 minutes and a standard deviation of 4 minutes. Suppose it is known that trip times have a normal distribution. The P-value for the appropriate test of significance to test the student's suspicion is:

(A) between 0.01 and 0.02.
(B) between 0.02 and 0.025.
(C) between 0.025 and 0.05.
(D) between 0.05 and 0.10.
(E) between 0.10 and 0.15.

12. We conduct a hypothesis test of $\mathrm{H}_0 : \mu = 20$ vs. $\mathrm{H}_\mathrm{a} : \mu \neq 20$ for the mean of some population at the 5% level of significance. Using a sample of size 30, we determine that H_0 should be rejected if $\bar{x} < 17$ or $\bar{x} > 23$. The power of the test against the alternative $\mathrm{H}_\mathrm{a} : \mu = 25$ is calculated to be 0.8621.

    Which of the following statements is **true**?

    (A) If we had instead used a sample size of 50, the probability of a Type II error would increase.

    (B) If we used a 5% level of significance and a sample of size 50, the power of the test would decrease.

    (C) The power of the test against the alternative $\mathrm{H}_\mathrm{a} : \mu = 15$ is 0.1379.

    (D) If we had instead used a 1% level of significance, the probability of a Type II error would decrease.

    (E) If we had instead used a 10% level of significance, the power of the test would increase.

13. Bottles of a certain brand of apple juice are supposed to contain 300 ml of juice. There is some variation from bottle to bottle because of imprecisions in the filling machinery. A consumer advocate inspector selects a random sample of 36 bottles and tests the hypotheses $\mathrm{H}_0 : \mu = 300$ vs. $\mathrm{H}_\mathrm{a} : \mu < 300$ at the 10% level of significance. Fill volumes are known to follow a normal distribution with standard deviation 3 ml. What is the power of the test if the true mean fill volume is actually 298 ml?

    (A) 0.9925 (B) 0.9943 (C) 0.9957 (D) 0.9967 (E) 0.9977

The next **two** questions (**14** and **15**) refer to the following:

Eight males and eight females volunteered to be part of an experiment. All 16 people were Caucasian and between the ages of 18 and 24. Each of the eight male participants was randomly assigned a night club, and each of the eight females was randomly assigned to one of these same eight night clubs. One Friday night, all 16 people went out to the bar. Each person then went up to the bar to order a drink (within 5 minutes of one another, with the order randomly determined.) The time (in seconds) until each customer was served was recorded. The table below contains some information that may be helpful:

Females	Males	Difference (d = Female − Male)
mean = 48	mean = 62	mean = −14
std. dev. = 18	std. dev. = 23	std. dev. = 15

The question of interest in this experiment is whether females receive faster service at bars on average than males. Assume the appropriate normality assumptions are satisfied.

14. What are the hypotheses for the appropriate test of significance?

(A) $H_0 : \mu_d = 0$ vs. $H_a : \mu_d < 0$

(B) $H_0 : \bar{X}_d = 0$ vs. $H_a : \bar{X}_d > 0$

(C) $H_0 : \mu_F = \mu_M$ vs. $H_a : \mu_F > \mu_M$

(D) $H_0 : \bar{X}_d = 0$ vs. $H_a : \bar{X}_d < 0$

(E) $H_0 : \mu_d = 0$ vs. $H_a : \mu_d > 0$

15. What is the P-value for the appropriate test of significance?

(A) between 0.005 and 0.01

(B) between 0.01 and 0.02

(C) between 0.02 and 0.025

(D) between 0.025 and 0.05

(E) between 0.05 and 0.10

The next **two** questions (**16** and **17**) refer to the following:

The following data represent the fat content found in samples of two popular brands of ice cream:

Brand	Fat					sample mean	sample variance
A	6.3	5.1	6.8	6.9	7.4	6.50	0.77
B	6.3	5.7	5.9	6.4	5.1	5.88	0.27

16. We would like to conduct a hypothesis test to determine whether the true mean fat content differs for the two brands of ice cream. Assuming fat content follows a normal distribution for both brands, we should use:

(A) a matched pairs t test with 4 degrees of freedom.

(B) a pooled two-sample t test with 8 degrees of freedom.

(C) a conservative two-sample t test with 7 degrees of freedom.

(D) a pooled two-sample t test with 9 degrees of freedom.

(E) a conservative two-sample t test with 8 degrees of freedom.

17. The P-value for the appropriate test of significance is calculated to be 0.21. We interpret this value to mean:

(A) If the true mean fat content was equal for the two brands, the probability of incorrectly concluding that the means differ would be 0.21.

(B) The probability that the true mean fat content is equal for the two brands is 0.21.

(C) The probability that the true mean fat content differs for the two brands is 0.21.

(D) If the true mean fat content were equal for the two brands, the probaility of observing a difference in sample means at least as extreme as 0.62 would be 0.21.

(E) If the true mean fat content differed for the two brands, the probability of observing a difference in sample means at least as extreme as 0.62 would be 0.21.

18. The distances (in km) travelled by random samples of University of Manitoba and University of Winnipeg students each day (from home to their university) are recorded and some summary statistics are shown below:

	n	$\bar{x}$	s
U of M	11	9.6	3.2
U of W	7	5.5	2.4

Distances are known to follow normal distributions for students at both universities. We would like to conduct a hypothesis test to determine whether U of M students travel further on average than U of W students. The P-value for the appropriate test of significance is:

(A) between 0.01 and 0.02

(B) between 0.02 and 0.025

(C) between 0.025 and 0.05

(D) between 0.05 and 0.10

(E) between 0.10 and 0.15

19. We take independent simple random samples from two normally distributed populations. Some summary statistics are shown in the table below:

Sample	Sample Size	Sample Mean	Sample Std. Dev.
1	12	43	16
2	7	37	9

We would like to construct a 90% confidence interval for the difference in the two population means $\mu_1 - \mu_2$. The margin of error for the confidence interval is:

(A) 5.74 (B) 6.63 (C) 7.65 (D) 9.98 (E) 11.53

20. We would like to conduct a pooled two-sample t test to compare the means of two populations. Which of the following assumptions are necessary?

(I)	Both populations are normally distributed.
(II)	The two samples are dependent.
(III)	The two samples are independent.
(IV)	The population standard deviations are known.
(V)	The population standard deviations are equal.
(VI)	The sample standard deviations are equal.

(A) III, IV and V

(B) I, III, V and VI

(C) I, III and VI

(D) I, II, IV and VI

(E) I, III and V

21. In which of the following situations are the matched pairs t procedures appropriate?

(I)	We would like to compare the mean number of mosquitoes caught in two different types of mosquito traps. A sample of 20 locations is selected in Winnipeg, and one of each type of trap is placed at each location.
(II)	We would like to compare the mean commuting times from home to work for workers in Winnipeg and Toronto. A sample of 30 workers in each city is selected.
(III)	We would like to compare the mean summer temperatures in Winnipeg and Brandon (Manitoba's second largest city, 200 km west of Winnipeg). On a sample of 25 days, we measure the temperature in each city.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

22. The serum calcium measurements for samples of two breeds of cows are shown below. Serum calcium levels are known to follow a normal distribution for both breeds.

Chester White	116	112	82	63	117	69
Hampshire	62	79	80	85	60	71

We would like to conduct a hypothesis test to determine whether the true mean serum calcium level differs for the two breeds. Some *JMP* output that may be helpful is shown below:

Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
Hampshire	6	72.8333	10.2258	4.175	62.102	83.56
Chester White	6	93.1667	24.7501	10.104	67.193	119.14

t Test

Chester White-Hampshire

Assuming equal variances

Difference	20.333	t Ratio	
Std Err Dif	10.933	DF	10
Upper CL Dif	44.693	Prob > |t|	
Lower CL Dif	-4.026	Prob > t	
Confidence	0.95	Prob < t	

t Test

Chester White-Hampshire

Assuming unequal variances

Difference	20.333	t Ratio	
Std Err Dif	10.933	DF	6.658692
Upper CL Dif	46.456	Prob > |t|	
Lower CL Dif	-5.789	Prob > t	
Confidence	0.95	Prob < t	

The P-value for the appropriate test of significance is:

(A) between 0.01 and 0.02

(B) between 0.025 and 0.05

(C) between 0.05 and 0.10

(D) between 0.10 and 0.20

(E) between 0.20 and 0.30

23. A student would like to conduct a hypothesis test to determine whether the true mean exam score for students in Dr. E. Zee's class is greater than that for student's in Dr. D. Ficult's class. Exam scores are known to follow a normal distribution for each of the two classes. The student selects random samples of students from each professor's class. Some summary statistics for their exam scores are shown below:

Group	Sample Size	Sample Mean	Sample Std. Dev.
Dr. E. Zee	14	74	6
Dr. D. Ficult	10	53	14

What is the value of the test statistic for the appropriate test of significance?

(A) 3.77 (B) 4.46 (C) 5.04 (D) 6.62 (E) 7.13

24. A horticulturist is investigating the phosphorous content of tree leaves from three different varieties of apple trees. Phosphorous content is known to follow a normal distribution for each of the three varieties. Random samples of four leaves from each of the three varieties are analyzed for phosphorous content. Population variances are known to be equal for all three varieties. We would like to test the hypothesis of equal population means for the three varieties at the 10% level of significance. The critical value for the appropriate hypothesis test is:

(A) 1.36 (B) 2.81 (C) 3.01 (D) 5.22 (E) 9.38

25. We take simple random samples from two normally distributed populations and measure the value of some variable X. We conduct a pooled two-sample t test to determine whether there is evidence that the population means differ. We calculate a test statistic of $t = 2.47$ and a P-value of 0.04. Suppose we had instead conducted the test using an analysis of variance. What would be the value of the test statistic and the P-value?

(A) 6.10 and 0.04
(B) 2.47 and 0.0016
(C) 1.57 and 0.04
(D) 6.10 and 0.0016
(E) 2.47 and 0.20

26. We are conducting an analysis of variance to compare the means of four populations. In conducting this test, we will commit a Type I error if we:

(A) fail to conclude that all four means differ when in fact this is the case.

(B) conclude that at least one of the means differs when in fact they are all equal.

(C) conclude that all four means differ when in fact they are all equal.

(D) fail to conclude that at least one mean differs when in fact this is the case.

(E) conclude that one mean differs when in fact all four means differ.

27. We would like to compare the means of three normally distributed populations with common variance. We conduct three separate pooled two-sample t tests and we obtain the P-values shown below:

Test 1	Test 2	Test 3
H_0: $\mu_1 = \mu_2$	H_0: $\mu_1 = \mu_3$	H_0: $\mu_2 = \mu_3$
H_a: $\mu_1 \neq \mu_2$	H_a: $\mu_1 \neq \mu_3$	H_a: $\mu_2 \neq \mu_3$
P-value = 0.02	P-value = 0.23	P-value = 0.11

Suppose we were to conduct an analysis of variance F test at the 5% level of significance to compare the three means. Which of the following statements is true?

(A) The P-value of the ANOVA F test would be 0.02 (the smallest of the P-values for the three two-sample tests).

(B) The P-value of the ANOVA F test would be 0.12 (the average of the P-values for the three two-sample tests).

(C) The null hypothesis would be rejected, since one of the three two-sample tests had a P-value less than 0.05.

(D) The null hypothesis would not be rejected, since two of the three two-sample tests had a P-value greater than 0.05.

(E) There is no way to determine the results of the ANOVA F test from the three two-sample tests. If we wanted to compare all three means, we should have conducted an analysis of variance test to begin with.

The next **three** questions (**28** to **30**) refer to the following:

A medical researcher wants to compare three different techniques designed to lower blood pressure in patients with hypertension. Five subjects are assigned to each of the three groups. The first group exercises, the second group meditates and the thrid group takes medication. After five weeks, the reduction in each person's blood pressure is recorded. The researcher wants to determine if there is any difference in the effectiveness of the three techniques. The ANOVA table (with some values missing) is shown below:

Source of Variation	*df*	Sum of Squares	Mean Squares	F
Groups		102		
Error				
Total		282		

28. What is the P-value of the test?

(A) less than 0.001
(B) between 0.001 and 0.01
(C) between 0.01 and 0.025
(D) between 0.025 and 0.05
(E) between 0.05 and 0.10

29. What is the estimate of the common variance for the three treatments?

(A) 4 (B) 7 (C) 13 (D) 15 (E) 51

30. We would like to construct a 95% confidence interval for the difference in the mean response for the exercise and medication groups. The margin of error for the appropriate confidence interval is:

(A) 5.25 (B) 5.34 (C) 5.65 (D) 9.53 (E) 12.49

Answer Key

1. C
2. A
3. A
4. D
5. E
6. A
7. C
8. E
9. D
10. B
11. D
12. E
13. D
14. A
15. B
16. B
17. D
18. C
19. E
20. E
21. E
22. D
23. B
24. C
25. A
26. B
27. E
28. E
29. D
30. B