SAMPLE FINAL 1 – Part B

1.

Game 1

Game 2	.2 F	.8 C
.3 U	FU	CU
.7 S	FS	CS

Game 3	.2 .3 FU	CU	FS	CS
.4 G	FUG	CUG	FSG	CSG
.6 J	FUJ	CUJ	FSJ	CSJ

1.(a)

Outcome	FUG	CUG	FSG	CSG	FUJ	CUJ	FSJ	CSJ
	$.2\times.3\times.4$	$.8\times.3\times.4$						
Probability	.024	.096	.056	.224	.036	.144	.084	.336

1.(b)

X	0	1	2	3
Probability	.144	.468	.332	.056

"X=0" → CUJ $\qquad P(X=0)=.144$

"X=1" → CUG, FUJ, CSJ $\qquad P(X=1)=.468$

.096 + .036 + .336

"X=2" → FUG, CSG, FSJ $\qquad P(X=2)=.332$

.024 + .224 + .084

"X=3" → FSG $\qquad P(X=3)=.056$

1.(c) Expected Value $= \mu = \sum x\, p(x)$

Variance $= \sigma^2 = \sum (x-\mu)^2 p(x)$

$$\mu = 0\times.144 + 1\times.468 + 2\times.332 + 3\times.056$$

$$\boxed{\mu = 1.3}$$

$$\begin{aligned}\sigma^2 = {} & (0-1.3)^2\times.144 \\ & + (1-1.3)^2\times.468 \\ & + (2-1.3)^2\times.332 \\ & + (3-1.3)^2\times.056 \\ \hline & 0.61\end{aligned}$$

$$\boxed{\sigma^2 = 0.61}$$

1.(d) Conditional Probability!

Given 2 Europeans won

What is probability Finland won?

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Given 2 Europeans won

Outcome	✓ FUG	CUG	FSG	✓ CSG	FUJ	CUJ	✓ FSJ	CSJ
Probability	.024	.096	.056	.224	.036	.144	.084	.336

$P(F \mid 2 \text{ Europeans won})$

$$= \frac{.024 + .084}{.024 + .224 + .084} = \frac{.108}{.332} = \boxed{.3253}$$

2. Males $n_1 = 300$ $x_1 = 180$ $\hat{p}_1 = \frac{180}{300} = 0.6$

Females $n_2 = 200$ $x_2 = 104$ $\hat{p}_2 = \frac{104}{200} = .52$

2. (a) $(\hat{p}_1 - \hat{p}_2) \pm z^* \, SE(\hat{p}_1 - \hat{p}_2)$

$z^* = 1.96$ for 95% confidence

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$= \sqrt{\frac{.6(1-.6)}{300} + \frac{.52(1-.52)}{200}}$$

$$SE(\hat{p}_1 - \hat{p}_2) = 0.045254834\ldots$$

Confidence Interval is

$$(.6 - .52) \pm 1.96(0.0452\ldots)$$

$$.08 \pm 0.0887$$

$$\boxed{(-.0087, .1687)}$$

2. (b) We are 95% confident the difference in proportions between male and female voters is between $-.0087$ and $.1687$. If we repeatedly took samples of the same size and computed a 95% confidence interval for the difference, then, in the long run, 95% of the intervals we make will contain the true difference in proportions.

2. (c) $H_0: p_1 = p_2$ vs $H_a: p_1 \neq p_2$

Test statistic $z = \dfrac{\hat{p}_1 - \hat{p}_2}{SE(\hat{p}_1 - \hat{p}_2)}$

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

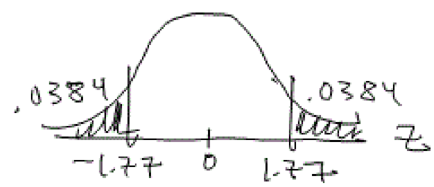
where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2} = \dfrac{180 + 104}{300 + 200} = \dfrac{284}{500}$

$$\hat{p} = .568$$

$$SE(\hat{p}_1 - \hat{p}_2) = \sqrt{.568\,(1 - .568)\left(\frac{1}{300} + \frac{1}{200}\right)} = .045219464$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE(\hat{p}_1 - \hat{p}_2)} = \frac{.6 - .52}{.045219464} \Rightarrow z = 1.769\ldots$$

test statistic $z = 1.77$



$\boxed{\text{P-value} = 2 \times .0384 = .0768}$

Do not reject H_0 (P-value not less than 5%)

There is no statistically significant evidence of a difference in the proportions of males and females who support the candidate.

2. (d) Assuming the proportions are the same for male and female supporters, there is a .0768 probability that we would get a test statistic of −1.77 or lower or 1.77 or higher.

<u>Generic P-value interpretation</u>

Assuming ______ (H₀ is correct),

there is a ______ (P-value) probability

that we would get a test statistic

of ______ (test statistic) or ______ higher/lower (shaded region).

3. A 2-tailed, 2 proportion z test is equivalent to doing a chi-square test for homogeneity (or independence)

where test statistic is $z^2 = \chi^2$

H_0: The distributions for male and female supporters are homogeneous
H_a: The distributions are not homogeneous

test statistic $= \chi^2 = z^2 = (1.77)^2 = 3.13$

p-value $= .0768$ (as found in #2)

Do not reject H_0. Same conclusion as in 2.(c)