SAMPLE EXAM 2 – Part B

1. (a) H_0: The employee is telling the truth.

H_a: The employee is lying.

Type I error: Rejecting H_0 when H_0 is true
In this context:

Calling the employee a liar, when he/she is telling the truth.

Type II error is deciding the employee is telling the truth when they are actually lying.

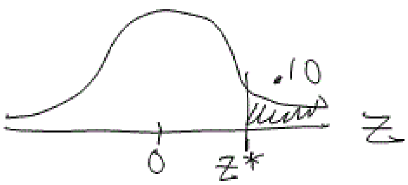
The consequences of ~~the~~ Type I error would be accusing your employee of theft when they are not a thief. Probably firing an employee unfairly.

Consequences of a Type II error would be probably keeping an employee who is a thief.

1.(b) $H_0: \mu = 2000$ vs $H_a: \mu > 2000$

Given $\alpha = 10\%$, $\sigma = 100$, $n = 36$

Get the $\bar{x}$ decision rule!



$z^* = 1.282$

$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = 1.282\left(\frac{100}{\sqrt{36}}\right) + 2000 = 2021.3367$$

Reject H_0 if $\bar{x} > 2021.3667$

α	β
Reject H_0 if $\bar{x} > 2021.3667$	Don't reject H_0 if $\bar{x} \le 2021.3667$
$\mu_0 = 2000$	$\mu_a = 2050$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{2021.3667 - 2050}{100/\sqrt{36}}$$

$z = -1.72$

Table A: $\beta = .0427$

Power $= 1 - \beta = 1 - .0427$

$\boxed{\text{Power is } .9573}$

2.

# of females	# of shows	
0	50	$0 \times 50 = 0$
1	94	$1 \times 94 = 94$
2	52	$2 \times 52 = 104$
3	4	$3 \times 4 = 12$
TOTALS	200	210

If this is Binomial, there has to be n and p. We know $X = 0, 1, 2 \ldots n$ since X stops counting at 3, we know $\boxed{n=3}$. We are not given p, so we use the sample to estimate it.

$$\hat{p} = \frac{210}{3 \times 200} = \frac{210}{600} = .35$$

H_0: The distribution is binomial with $n=3$ and $p=.35$.

H_a: The distribution is not binomial with $n=3$ and $p=.35$.

# of females	Obs. Count	Exp. Count	χ^2
0 (.2746)	50	54.92	$\frac{(50-54.92)^2}{54.92} = 0.44$
1 (.4436)	94	88.72	0.31
2 (.2389)	52	47.78	0.37
3 (.0429)	4	8.58	2.44
Found on Table C	200	200	$\boxed{\chi^2 = 3.56}$ test statistic

$$df = 4 - 1 - 1 = 2$$

because we estimated the parameter "p"

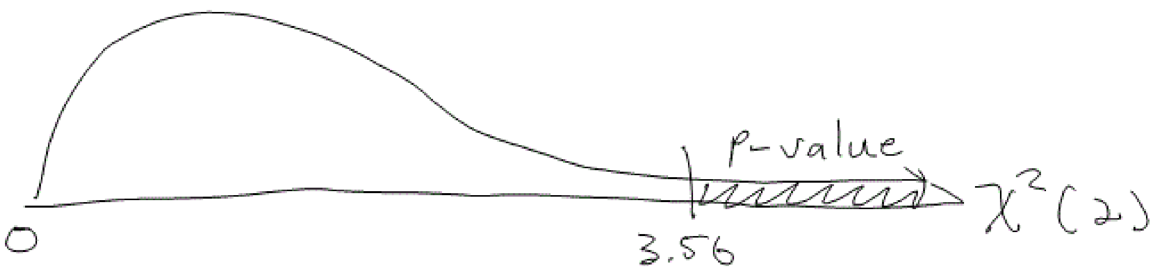


Table F: P-value is between .15 and .20

$\alpha = 5\%$ $\longrightarrow$ Do not reject H_0

There is insufficient evidence to reject the claim that this is Binomial with $n=3$, $p=.35$.

3. (a) $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

where x = weight of car (in 1000s of pounds)
y = fuel efficiency (in mpg)

β_0 is the true intercept
β_1 is the true slope
ε_i is the residual for any observation (x_i, y_i)

3. (b) Since the residual plot has no pattern, it is reasonable to assume that x and y have a linear relationship. The regression model is valid.

3. (c) $b \pm t^* SE_b$

Given $\hat{y} = 51.47 - 8.89x \rightarrow b = -8.89$
$n = 10$ observations $\underline{df = 8}$
$95\% \longrightarrow t^* = 2.306$

$$SE_b = \frac{s_e}{\sqrt{(x_i - \bar{x})^2}} \quad \text{where } s_e = \sqrt{\frac{\Sigma(y_i - \hat{y})^2}{n-2}}$$

$$s_e = \sqrt{\frac{24.92}{8}} = 1.7649$$

$$SE_b = \frac{s_e}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{1.7649}{\sqrt{3.8025}} = \underline{0.9051}$$

$$b \pm t^* SE_b = -8.89 \pm 2.306(0.9051)$$
$$= -8.89 \pm 2.09$$

$\boxed{(-10.98, -6.80)}$

3.(d) We are 95% the true slope is somewhere between −10.98 and −6.80. If we repeatedly took samples of size 10, and made 95% confidence intervals for the slope, in the long run, 95% of the intervals will contain the true slope.

3.(e) $H_0: \beta_1 = 0$ vs $H_a: \beta_1 \neq 0$ $df = 8$

$$t = \frac{b}{SE_b} = \frac{-8.89}{0.9051} = -9.822 \text{ (test statistic)}$$

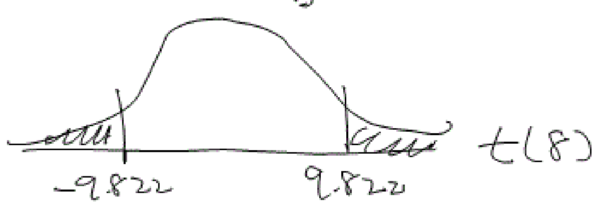


Table D → Upper Tail is between 0 and .0005

$\boxed{\text{P-value is between } 0 \text{ and } .001}$

If $\alpha = 5\% \longrightarrow$ Reject H_0

There is statistically significant evidence that $\beta_1 \neq 0$. There is significant evidence of a linear relationship between the weight of a car and its fuel efficiency.

3.(f) Assuming there is no linear relationship between weight and fuel efficieny, there is somewhere between a 0 and .001 probability that we would get a test statistic of −9.822 or lower or 9.822 or higher.

3. (g) $\hat{y} \pm t^* SE_{\hat{y}}$

$\hat{y} = 51.47 - 8.89x$ given $x = 3200$ pounds

$x = 3.2$ thousand pounds

$\hat{y} = 51.47 - 8.89(3.2) = 23.022$

$df = 8$, $t^* = 2.306$ for 95% confidence

$$SE_{\hat{y}} = s_e \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$$

$$= 1.7649 \sqrt{1 + \frac{1}{10} + \frac{(3.2 - 3.01)^2}{3.8025}}$$

$SE_{\hat{y}} = 1.859$

$\hat{y} \pm t^* SE_{\hat{y}} = 23.022 \pm 2.306(1.859)$

23.022 ± 4.287

$$\boxed{(18.735,\ 27.309)}$$