

Sample Midterm Test – A

1. We would like to conduct a hypothesis test to examine whether there is evidence that the true mean amount spent on textbooks by a U of M student in one semester differs from \$400. A random sample of 50 students is selected and the mean amount they spent on textbooks for one semester is calculated to be \$430. Assume the population standard deviation is known to be \$165. What is the P-value for the appropriate hypothesis test?

- (A) 0.1970 (B) 0.1112 (C) 0.0630 (D) 0.0985 (E) 0.1260

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2. We take a simple random sample of 16 adults and ask them how long they sleep on a typical night. The sample mean is calculated to be 7.2 hours. Suppose it is known that the number of hours adults sleep at night follows a normal distribution with standard deviation 1.8 hours. An 88% confidence interval for the true mean time adults sleep at night is:

(A) (6.80, 7.60)

(B) (6.30, 8.10)

(C) (6.60, 7.80)

(D) (6.70, 7.70)

(E) (6.50, 7.90)

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3. The Manitoba Department of Agriculture would like to estimate the average yield per acre of a new variety of corn for farms in southwestern Manitoba. It is desired that the final estimate be within five bushels per acre of the true mean yield. Due to cost restraints, a sample of no more than 65 one-acre plots of land can be obtained to conduct an experiment. The population standard deviation is known to be 17.33 bushels per acre. What is (approximately) the maximum confidence level that could be attained for a confidence interval that meets the Agriculture Department's specification?

(A) 90%

(B) 95%

(C) 96%

(D) 98%

(E) 99%

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4. A hypothesis test is conducted at significance level α to determine whether the mean of some population is greater than some value μ_0 . Then $1 - \alpha$ is equal to:
- (A) the probability of rejecting H_0 when H_0 is true.
 - (B) the probability of failing to reject H_0 when H_0 is true.
 - (C) the probability of rejecting H_0 when H_a is true.
 - (D) the probability of failing to reject H_0 when H_a is true.
 - (E) the probability that H_a is true.

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5. A simple random sample of six male patients over the age of 65 is being used in a blood pressure study. The standard error of the mean blood pressure of these six men was calculated to be 7.8. What is the standard deviation of these six blood pressure measurements?

(A) 2.8

(B) 3.2

(C) 14.4

(D) 19.1

(E) 46.8

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The next **two** questions (**6** and **7**) refer to the following:

The systems department of the Texaco Oil Company runs a large number of simulation programs on their mainframe computer. A manufacturer of new simulation software claims their program will run the simulation faster than the current mean of 30 minutes. The new program will be run 20 times. A hypothesis test of $H_0 : \mu = 30$ vs. $H_a : \mu < 30$ is to be conducted, and it is decided that H_0 will be rejected if $\bar{x} \leq 28.3$ minutes. Run times for the new program are known to follow a normal distribution with standard deviation 3.7 minutes.

6. The level of significance of the test is closest to:

(A) 0.01

(B) 0.02

(C) 0.03

(D) 0.04

(E) 0.05

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7. What is the probability of making a Type II error if the true mean run time for the new program is actually 27 minutes?

- (A) 0.0192 (B) 0.0244 (C) 0.0384 (D) 0.0475 (E) 0.0582

8. In which of the following situations is a Type I error more serious than a Type II error?
Suppose you are deciding whether or not to:

- (I) Jump from an airplane
 H_0 : Parachute will not open
 H_a : Parachute will open
- (II) Take a cheat sheet into an exam
 H_0 : Will not get caught
 H_a : Will get caught
- (III) Walk across the Red River in January
 H_0 : Ice will not break
 H_a : Ice will break

- (A) I only
- (B) III only
- (C) I and II only
- (D) II and III only
- (E) I, II and III

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9. We take random samples of individuals from two normally distributed populations and record the value of some random variable X . Some summary statistics are shown below:

Population	Sample Size	Sample Mean	Sample Std. Dev.
1	7	29	4.0
2	14	25	9.2

We would like to construct a 95% confidence interval for the difference $\mu_1 - \mu_2$ in population means. What is the value of the standard error of $\bar{X}_1 - \bar{X}_2$?

- (A) 2.24 (B) 1.86 (C) 3.27 (D) 2.89 (E) 3.45

The next **three** questions (**10**, **11** and **12**) refer to the following:

Coke and Pepsi are the two most popular brands of cola on the market. Do consumers prefer either one of the two brands of cola over the other? We conduct an experiment as follows: 20 volunteers participate in a blind taste test. Each volunteer tastes both Coke and Pepsi (in random order) and scores the taste of each cola on a scale from 0 to 100. Some information that may be helpful is shown in the table below:

Scores for Coke	Scores for Pepsi	Difference (Coke – Pepsi)
mean = 78	mean = 83	mean = -5
std. dev. = 27	std. dev. = 24	std. dev. = 13

10. Which of the following statements is/are **true**?

- (I) The scores for Coke and Pepsi for each individual are independent.
- (II) The scores for Coke and Pepsi for each individual are dependent.
- (III) In order to conduct the matched pairs t test, we must assume that scores for Coke and scores for Pepsi both follow normal distributions.
- (IV) In order to conduct the matched pairs t test, we must assume that the difference in scores (Coke – Pepsi) follow a normal distribution.

- (A) I only
- (B) I and III only
- (C) I and IV only
- (D) II and III only
- (E) II and IV only

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11. We will use the critical value method to conduct the test, with a 10% level of significance. Assuming the appropriate assumptions are satisfied, the rejection rule is to reject H_0 if:

- (A) $|t| \geq 1.328$ (B) $t \geq 1.729$ (C) $|t| \geq 1.734$ (D) $t \geq 1.328$ (E) $|t| \geq 1.729$

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12. Assuming the appropriate assumptions are satisfied, what is the value of the test statistic for the appropriate test of significance?

(A) -0.38

(B) -7.69

(C) -2.85

(D) -1.72

(E) -0.88

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The next two questions (13 and 14) refer to the following:

Several workers in a large office building suspect that coffee vending machine A dispenses more coffee on average than vending machine B. To test this belief, several samples were taken from each machine. The amounts (in ounces) dispensed and some summary statistics are given below:

Machine	n	\bar{x}	s
A	12	11.74	0.56
B	8	11.18	0.79

13. Assuming fill volumes follow a normal distribution for both machines, the test statistic for the appropriate test of significance is:

(A)
$$\frac{11.74 - 11.18}{\sqrt{0.46 \left(\frac{1}{12} + \frac{1}{8} \right)}}$$

(D)
$$\frac{11.74 - 11.18}{\sqrt{\frac{(0.56)^2}{11} + \frac{(0.79)^2}{7}}}$$

(B)
$$\frac{11.74 - 11.18}{\sqrt{\frac{(0.56)^2}{12} + \frac{(0.79)^2}{8}}}$$

(E)
$$\frac{11.74 - 11.18}{\sqrt{0.68 \left(\frac{1}{12} + \frac{1}{8} \right)}}$$

(C)
$$\frac{11.74 - 11.18}{\sqrt{0.43 \left(\frac{1}{12} + \frac{1}{8} \right)}}$$

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14. We would make a Type II error in this test if we concluded that:

- (A) Machine A dispenses more on average than Machine B when in fact Machine B dispenses more on average than Machine A.
- (B) there is no evidence that Machine A and B dispense the same amount on average when in fact Machine A does dispense more on average.
- (C) Machine B dispenses more on average than Machine A when in fact Machine A dispenses more on average than Machine B.
- (D) Machine A dispenses more on average than Machine B when in fact Machines A and B dispense the same amount on average.
- (E) there is no evidence that Machine A dispenses more on average than Machine B when in fact Machine A dispenses more on average.

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15. Two different alloys are being considered for making lead-free solder used in the wave soldering process for printed circuit boards. An important characteristic of solder is its melting point, which is known to follow a normal distribution. A study was conducted using a random sample of 15 pieces of solder made from an old alloy and 8 pieces of solder made from a new alloy. The temperature (in °C) at which each of the pieces melted was determined. We wish to test if the mean melting point of the new alloy is significantly higher than the old alloy. Some *JMP* output that may be helpful is shown below:

Means and Std Deviations						
Level	Number	Mean	Std Dev	Std Err		
				Mean	Lower 95%	Upper 95%
Old	15	215.21	5.27	1.36	212.30	218.13
New	8	220.00	2.27	0.80	218.10	221.90

t Test			
New-Old			
Assuming equal variances			
Difference	4.78667	t Ratio	
Std Err Dif	1.96756	DF	21
Upper CL Dif	8.87842	Prob > t	0.0240*
Lower CL Dif	0.69491	Prob > t	0.0120*
Confidence	0.95	Prob < t	0.9880

t Test			
New-Old			
Assuming unequal variances			
Difference	4.78667	t Ratio	
Std Err Dif	1.57836	DF	20.47771
Upper CL Dif	8.07415	Prob > t	0.0065*
Lower CL Dif	1.49918	Prob > t	0.0032*
Confidence	0.95	Prob < t	0.9968

What is the value of the test statistic for the appropriate test of significance?

- (A) 2.43 (B) 2.90 (C) 3.03 (D) 4.17 (E) 6.09

16. Random samples of contestants who have appeared on two popular game shows (*Wheel of Fortune* and *Jeopardy*) are selected. The *Wheel of Fortune* contestants in the sample won an average of \$5700 more than the *Jeopardy* contestants in the sample. A 95% confidence interval for the difference in the mean amount of money won by contestants on the two game shows is calculated to be $(-1900, 13300)$. We would like to conduct a hypothesis test to determine whether the mean amounts won on the two game shows differ. Assume the appropriate normality assumptions are satisfied. We would:
- (A) reject H_0 at the 5% level of significance since the value 0 is contained in the 95% confidence interval.
 - (B) fail to reject H_0 at the 10% level of significance since the value 0 is contained in the 95% confidence interval.
 - (C) fail to reject H_0 at the 5% level of significance since the value 5700 is contained in the 95% confidence interval.
 - (D) reject H_0 at the 10% level of significance since the value 5700 is contained in the 95% confidence interval.
 - (E) fail to reject H_0 at the 5% level of significance since the value 0 is contained in the 95% confidence interval.

The next **three** questions (**17 to 19**) refer to the following:

Battery life of MP3 players is of great concern to customers. A consumer group tested four brands of MP3 players to determine the battery life. Samples of players of each brand were fully charged and left to run on medium volume until the battery died. The following table displays the number of hours each of the batteries lasted:

	<u>Brand</u>			
	A	B	C	D
	24	26	18	27
	19	28	20	24
	22	24	21	22
		25		24
mean	21.67	25.75	19.67	24.25
std. dev.	2.52	1.71	1.53	2.06

We would like to conduct an analysis of variance to compare the performance of the four brands. Suppose that all necessary assumptions have been satisfied. The ANOVA table (with some values missing) is shown below:

Source of Variation	<i>df</i>	Sum of Squares	Mean Squares	<i>F</i>
Groups				
Error			3.88	
Total		113.71		

17. What is the alternative hypothesis for the appropriate test of significance?

- (A) H_a : All population variances are different.
- (B) H_a : At least one sample mean differs.
- (C) H_a : All population means are different.
- (D) H_a : At least one population variance differs.
- (E) H_a : At least one population mean differs.

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18. What is the P-value of the appropriate test of significance?

- (A) less than 0.001
- (B) between 0.01 and 0.025
- (C) between 0.025 and 0.05
- (D) between 0.05 and 0.10
- (E) greater than 0.10

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19. We would like to construct a 95% confidence interval for the true mean battery life for all Brand B MP3 players. The 95% confidence interval is:

(A) (23.85, 27.65)

(B) (23.17, 28.33)

(C) (23.03, 28.47)

(D) (22.51, 28.99)

(E) (22.04, 29.46)

The next two questions (20 and 21) refer to the following:

We conduct an experiment to compare the responses to six different treatments. Volunteers are randomly assigned to receive one of the six treatments. The response variable is measured for each individual and an analysis of variance F test is conducted to assess the equality of the means for the six treatments. Responses for each of the six treatments are known to follow normal distributions. Some summary statistics are shown below:

Treatment	Sample Size	Sample Mean	Sample Std. Dev.
1	3	17	8
2	5	14	7
3	2	21	11
4	4	20	6
5	3	18	9
6	4	12	10

20. Under the null hypothesis, we assume all population means are equal. What is the estimate of this common population mean?

- (A) 15.99 (B) 16.43 (C) 16.78 (D) 17.00 (E) 17.22

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21. What is the 5% critical value for the appropriate test of significance?

(A) 2.57

(B) 2.68

(C) 2.79

(D) 2.85

(E) 2.90

The next two questions (22 and 23) refer to the following:

We conduct an experiment to compare the responses to five different treatments. Volunteers are randomly assigned to receive one of the five treatments. Each treatment is assigned to the same number of individuals. The response variable is measured for each individual and an analysis of variance F test is conducted to assess the equality of the means for the five treatments. Responses for each of the five treatments are known to follow normal distributions with common standard deviation. The ANOVA table (with some values missing) is shown below:

Source of Variation	df	Sum of Squares	Mean Squares	F
Groups		74.52		
Error	60		5.78	
Total				

22. How many individuals received each of the five treatments?

- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

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23. One assumption required in conducting an ANOVA F test is that all population standard deviations are equal. The estimate of this common population standard deviation is:

(A) 2.40

(B) 4.32

(C) 5.78

(D) 18.63

(E) 33.41

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24. In a game of poker, you are dealt five cards from a deck of 52 cards. What is the probability that you get a flush (five cards of the same suit)?

(A) 0.0005

(B) 0.0010

(C) 0.0020

(D) 0.0030

(E) 0.0040

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25. A hockey player scores a goal in 43% of his games. His team wins 65% of their games. In 31% of the team's games, the player scores a goal and the team wins. What is the probability that the team wins if the player does not score a goal?

(A) 0.3595

(B) 0.4625

(C) 0.4985

(D) 0.5965

(E) 0.6345

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26. Consider two independent events A and B. Suppose it is known that $P(A) = 0.27$ and $P(A \text{ or } B) = 0.5474$. What is $P(B)$?

(A) 0.38

(B) 0.46

(C) 0.29

(D) 0.33

(E) 0.41

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The next **three** questions (27 to 29) refer to the following:

Suppose we have the following facts about customers buying alcohol at the Liquor Mart:

- 45% buy wine (W).
- 37% buy beer (B).
- 20% buy wine **and** beer.
- 6% buy wine **and** vodka (V).
- 5% buy beer **and** vodka.
- 46% buy beer **or** vodka.
- 2% buy wine and beer and vodka.

27. If we randomly select a customer, what is the probability that he or she buys vodka?

- (A) 0.14 (B) 0.15 (C) 0.16 (D) 0.17 (E) 0.18

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28. If we randomly select a customer, what is the probability that he or she buys **only** wine?

(A) 0.17

(B) 0.18

(C) 0.19

(D) 0.20

(E) 0.21

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29. If we know a customer buys wine, what is the probability that he or she also buys beer?

(A) 0.37

(B) 0.54

(C) 0.27

(D) 0.82

(E) 0.44

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30. We roll two fair dice. Let X be the minimum of the two numbers rolled. The probability distribution of X is shown below:

x	1	2	3	4	5	6
$P(X = x)$	11/36	9/36	7/36	5/36	3/36	1/36

What is the expected value of X ?

(A) 2.53

(B) 2.71

(C) 2.92

(D) 3.14

(E) 3.50

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31. The time it takes a student to drive to university in the morning follows a normal distribution with mean 28 minutes and standard deviation 4 minutes. The time it takes the student to drive home from university in the afternoon follows a normal distribution with mean 25 minutes and standard deviation 3 minutes. Morning and afternoon commuting times are known to be independent. What is the probability that it takes the student longer to drive to university than to drive home?

- (A) 0.5517 (B) 0.6664 (C) 0.7257 (D) 0.8707 (E) 0.9987

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32. When an archer shoots an arrow, he hits the bullseye on the target 78% of the time. Whether he hits the bullseye on any shot is independent of any other shot. If he shoots ten arrows, what is the probability he hits the target at least 8 times?

- (A) 0.2984 (B) 0.3185 (C) 0.4437 (D) 0.5689 (E) 0.6169

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33. A random variable X follows a binomial distribution with parameters $n = 3$ and p . What must be the value of the parameter p if we know that $P(X = 2) = P(X = 3)$?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) $\frac{3}{4}$

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34. The number of customers entering a bank follows a Poisson distribution with parameter $\lambda = 2$ per minute. What is the probability that exactly 15 customers enter the bank in the first ten minutes after it opens?

- (A) 0.0517 (B) 0.0623 (C) 0.0741 (D) 0.0859 (E) 0.0972

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35. A random variable X follows a Poisson distribution with parameter λ . We are conducting a hypothesis test of $H_0: \lambda = 5$ vs. $H_a: \lambda < 5$. We decide to reject the null hypothesis if $X \leq 2$. What is the power of the test if $\lambda = 1$?

(A) 0.1912

(B) 0.3679

(C) 0.5518

(D) 0.7358

(E) 0.9197